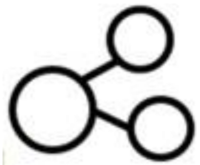
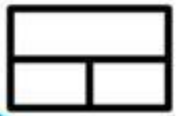
Calculation Policy

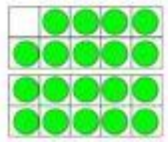
SUBTRACTION

Including Models and Images

October 2022







"The richest concept images will allow children to make the most effective numerical connections, enabling them to communicate mathematically."

Introduction

This policy exemplifies a recommended progression through both mental and written calculations for the four operations and has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of Mathematics. There is a strong focus on the use of models and images to support children's concept image of number and their understanding of how this relates to methods of calculation. It is also designed to give pupils a consistent and smooth progression of learning in calculations across the school, taking into account Maths No Problem! : a Singaporean teaching style in Maths.

The Calculation Policy shows methods that pupils will be taught within their respective year group. It is shown in teaching order. Children should be confident in choosing and using a strategy that they know will get them to the correct answer as efficiently as possible; pupils are free to choose their preferred method to solve calculations. Although our policy is set out based on The National Curriculum year group expectations, children work through the calculation policy systematically. Some children may therefore be working below year group expectation and should be taught the method appropriate for their individual stage in learning. This process should not be rushed; children should be moved on when they are ready.
"Children develop/learn in different ways and at different rates" – EYFS Principles.

Up to Year 3 the main emphasis should be on children working practically and mentally and recording through jottings. Once written methods are introduced, using practical images to support and develop mathematical understanding, mental skills must be kept sharp by continuing to develop and apply them with appropriate examples.

Should children be taught one standard method for each operation?

Children should work through the school's agreed progression in methods in order that they know and understand a compact standard method for each numerical operation by the end of Year 6.

How can children's readiness for written calculations be judged?

Judgements will need to be made as to whether pupils possess sufficient of these skills to progress. Different prerequisite skills are needed for each operation.

A short list of criteria for readiness for written methods of addition and subtraction would include:

- Do children know addition facts to 20?
- Do they understand place value and can they partition numbers into hundreds, tens and units?
- Do they use and apply commutative and associative laws of addition?
- Can they add at least three 1-digit numbers mentally?
- Can they add and subtract any pair of 2-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Corresponding criteria to indicate readiness to learn written methods for multiplication and division are:

- Do the children know their 2,3,4,5 and 10 times tables and corresponding division facts?
- Do they know the result of multiplying by 0 or 1?
- Do they understand 0 as a place holder?
- Can they multiply 2 and 3 digits mentally by 10 and 100?
- Can they use their knowledge of all the multiplication tables to approximate?
- Can they find products using multiples of 10?
- Do they use the commutative and associative laws of multiplication?
- Can they halve and double 2-digit numbers mentally?
- Can they use multiplication facts to derive mentally, other multiplication facts they don't know?
- Can they explain their mental strategies orally and record them using informal jottings?

Concrete, Pictorial, Abstract:

A key principle behind the Singapore Maths textbooks and Maths Mastery is based on the concrete, visual and abstract approach. Pupils are first introduced to an idea or skill by acting it out with real objects (a hands-on approach). Pupils then are moved onto the visual stage, where pupils are encouraged to relate the concrete understanding to pictorial representations. The final abstract stage is a change for pupils to represent problems by using mathematical notation.

Whilst this calculation policy aims to show the CPA approach to the different calculations, it is not always noted further up the year groups. However, it is expected that the CPA approach is used continuously in all new learning and calculations even when not noted.

Reasoning and Problem Solving:

Once children are fluent in the calculation strategy for their year group, we deepen and embed understanding through providing children with a range of reasoning and problem solving skills that allow the children to show their full understanding in a range of different context.

Monitoring of Written Calculations

It is important that procedures are in place to ensure that all staff are aware of the progression through calculations, and that children are being taught appropriate methods for their age and ability, which are in line with the agreed policy. This may include book sampling, reflective enquiries, monitoring of planning, learning walks and pupil interviews.

Progression for Subtraction

Key Vocabulary: take-away, subtract, minus, fewer, less, difference

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

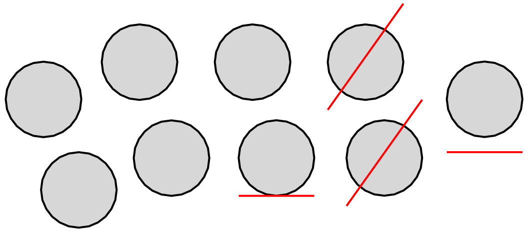
- the inverse of addition
- **not** commutative i.e. 5 - 3 is not the same as 3 - 5
- **not** associative i.e. 10 - 3 - 2 is not the same as 10 - (3 - 2)

EARLY LEARNING GOAL: Using quantities and objects, children subtract **two** single-digit numbers and **count on or back** to find the answer.

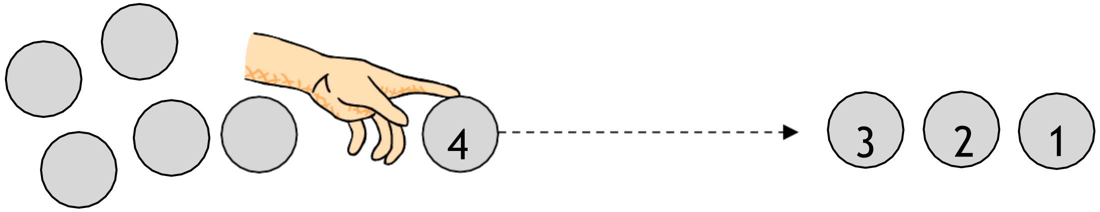
They should experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc. It is vital to develop a deep number sense so Number Talk is really important!

As mentioned in progression for Addition, Oral counting is a child's first experience of number and Mathematics:
- Making connections between saying the number names and counting objects is the first step towards children's understanding of the number system
- Counting is one tool for building up calculation strategies
- We need to count backwards is as well as forwards.

Taking away
Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g. 9 – 4.



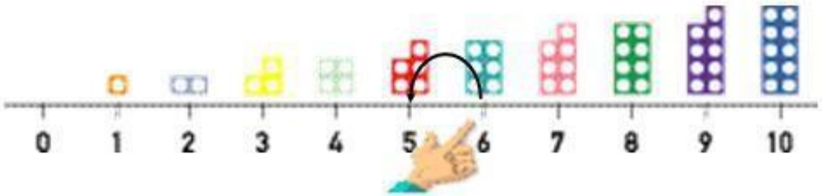
For illustration purposes, the amount being taken away are show crossed out. Children would be encouraged to physically remove these using touch counting.



By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

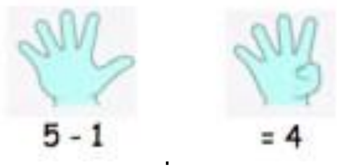
Those who are ready may record their own calculations

Children use number lines, tracks and numicon shapes to find one less, reinforcing the change in the shape of the number.

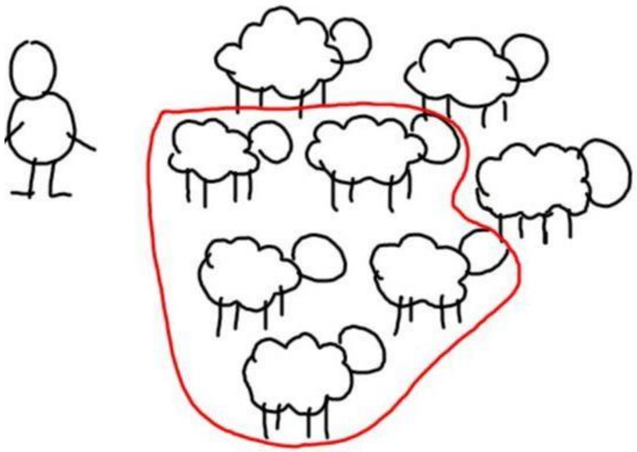


Counting back along a number line using a finger.

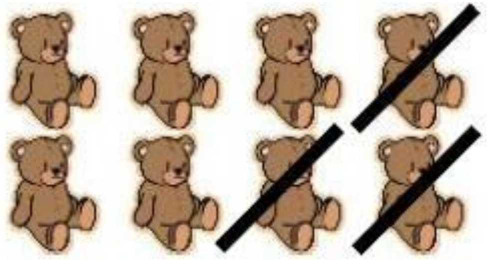
Children may begin to 'take away' by using their fingers.
Teachers provide the children with contexts for their subtraction to m



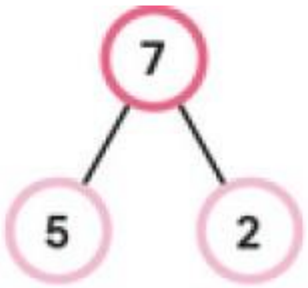
The farmer had 8 sheep. He lost 5. He now has 3 sheep.

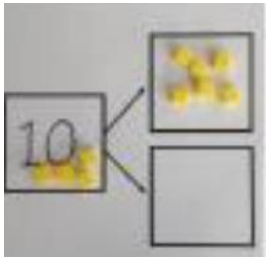


Lucy had 8 bears on her bed. Her little brother ran up and took 3 of the bears. She now has 5 bears.



After pupils have recognised different ways of making numbers, they should use this number bond knowledge to help with subtraction facts. They can use the bar model, place value grid and also the part part whole. The children are not expected to use these independently but are introduced to them in preparation for Year 1 and beyond:

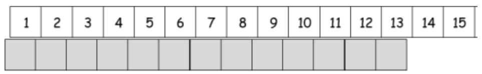
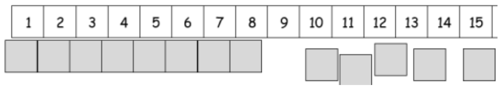
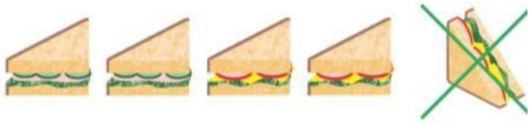


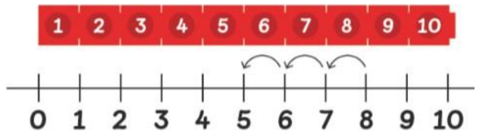


YEAR 1 - SUBTRACTION

End of Year Objective:

Subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

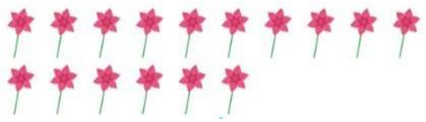
CONCRETE	PICTORIAL	ABSTRACT
To avoid the need to exchange for subtraction at this stage, we continue to use equipment such as counters, cubes and the Ones from the Base 10 equipment, but NOT the tens, e.g. 13 – 5 Touch count and remove the number to be taken away, in this case 4. Touch count to find the number that remains:   Multilink can be used against a number line: 8 – 3 = 5	Children will use images of the concrete objects explored practically as well as the bar model to show the calculations, including missing numbers, and the part whole model. Crossing out those that are being taken away: 7 – 2 = 5   How many sandwiches are left? 5 – 1 = 4 Use part part whole to support: 9 - 3 =	Children will use their knowledge of number to calculate and record a written calculation. 12 - 5 = 7 Children should be taught that the = sign does not always come at the end of the calculation. 16 = 9 + 7



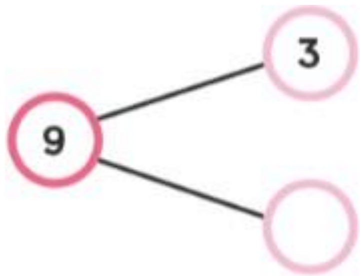
Number squares can also be used to support:

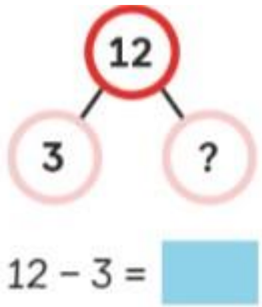
0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

16 – 4 =

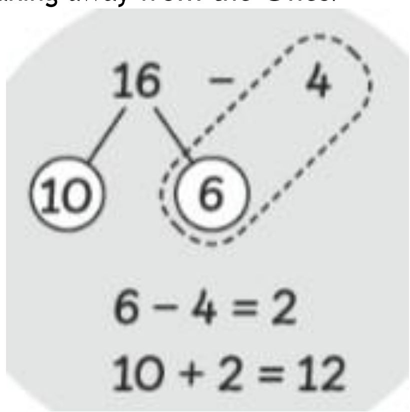


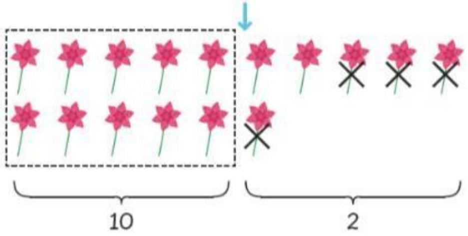
Recognising Ten and crossing out the Ones being subtracted from the Ones that are left:



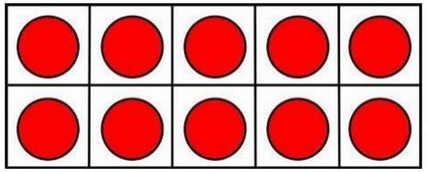


Taking away from the Ones:



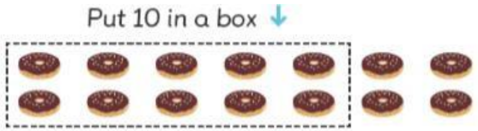


The Ten Frame supports this:

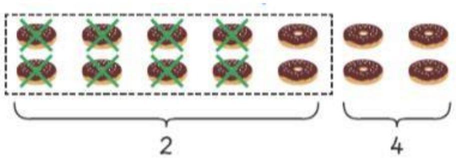


14 – 8 = 6

Recognise the Ten:

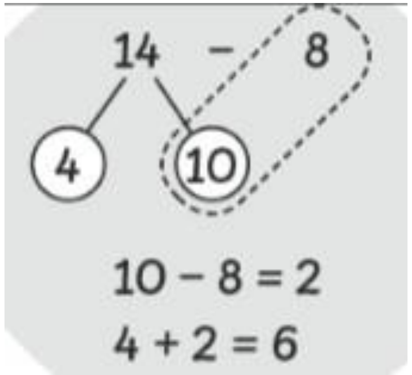


And then subtract the 8 from the 10:



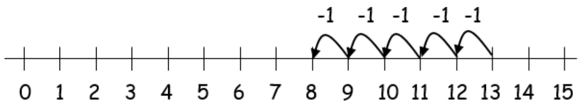
Learning fact families:

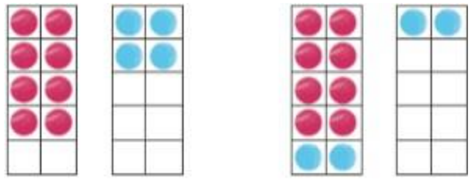
Taking away from the Ten:



Children can also use number lines:

13 – 5 = 8

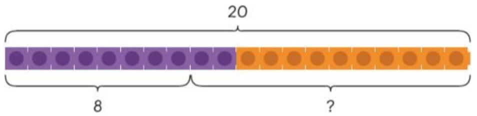




8 + 4 = 10 + 2

= 12

Bar model:



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

13 – 5 = 8

<u>YEAR 2 - SUBTRACTION</u>

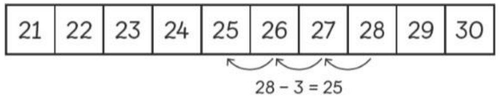
End of Year Objective:

Subtract a one digit number from a two-digit number and a two digit number from a two-digit number (including mentally).

CONCRETE

28 – 3 =

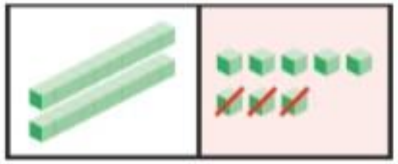
Counting back on a number board helps:



Using Base Ten:

28 – 3 =

Subtract 3 Ones from 8 Ones:

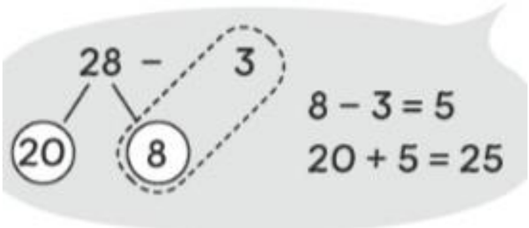


Subtract any Tens from the Tens:

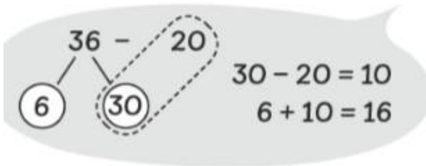
PICTORIAL

28 – 3 =

Partition with part part whole. Recognise the Tens and then subtract from the ones:



36 – 20 =

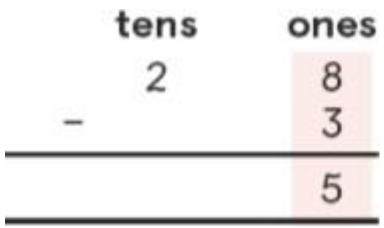


ABSTRACT

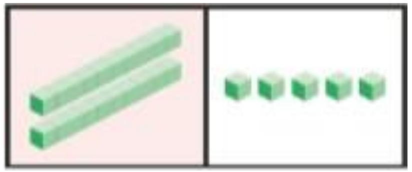
Without renaming:

28 – 3 =

Subtract the Ones, then subtract the Tens:



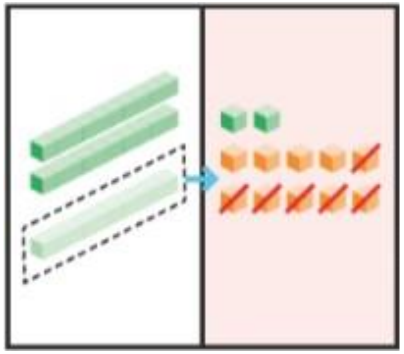
Use knowledge of subtraction number facts to take away groups of 10.



Renaming:

32 – 16 =

Regroup one Ten into ten Ones and then subtract the 6 Ones:



Then subtract the Ten:

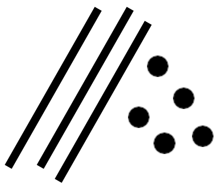
Use knowledge of number bonds to subtract using part part whole:

Children can record their drawings of Base 10 using slanted lines for the rods and dots for the ones blocks. Circling the tens and Ones that remain will help children to identify how many remain:

37 – 19 =

Children would cross out aTen and exchange for ten Ones:

STEP 1:



STEP 2:

4 - 1 = 3 therefore
40 – 10 = 30.

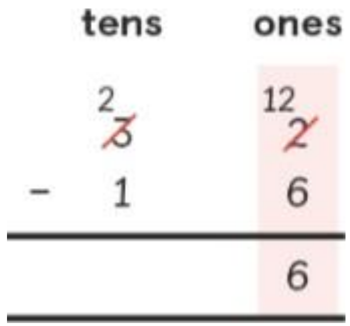
$5 - 2 = \boxed{}$

$50 - 20 = \boxed{}$

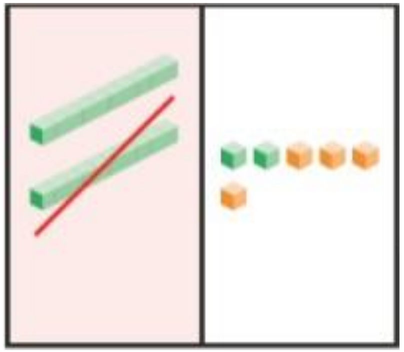
Renaming:

32 – 16 =

Exchange from the Tens to make 12 Ones

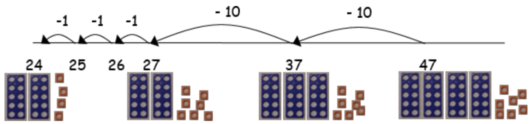


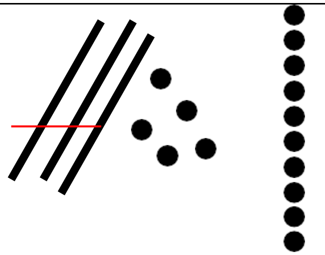
Then subtract the Ten from the two Tens:



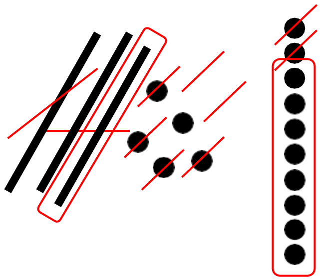
Supporting with a Number Line:

47 – 23 =





STEP 3:

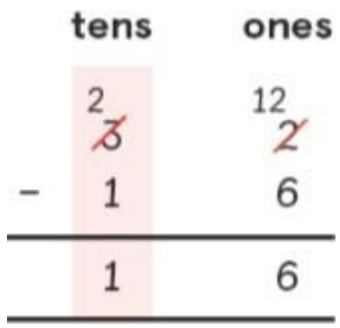


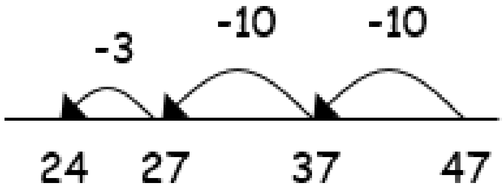
Circling the tens and Ones that remain will help children to identify how many remain.

Using a blank number line:

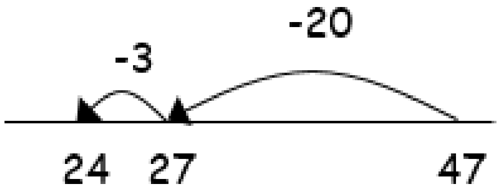
47 – 23 =

Children should become efficient in recognising 7 – 3 = 4 so:





And then efficiently combining the Tens in one jump:

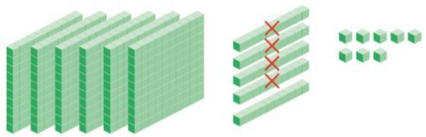
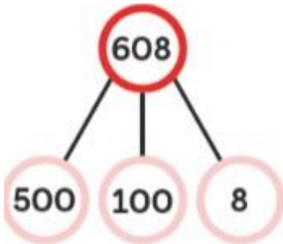


YEAR 3 - SUBTRACTION

End of Year Objective:

Subtract numbers with up to three digits, using formal written method of columnar subtraction.*

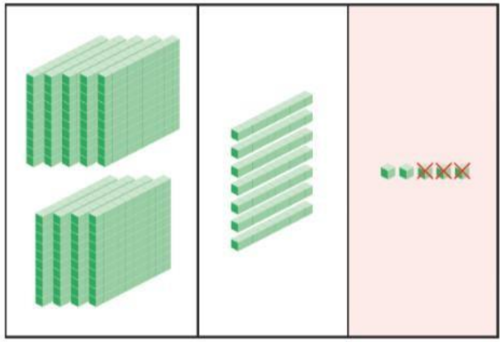
**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4 It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

CONCRETE	PICTORIAL	ABSTRACT
Children will build on their knowledge of using Base 10 equipment from Year 2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to represent the first number, removing the Ones and Tens as appropriate (as with the more informal method in Year 2). **Subtracting multiples of 10:** 658 – 40 = 	Children should continue to partition using part part whole:  Children continue to apply their knowledge of counting back in multiples of ten. They partition with part part whole:	By the end of year 3, children should also extend this method for three digit numbers. Expanded method (following on from arrow cards and Base Ten): $\begin{array}{rccc} & 60 & & \\ & \cancel{70} & \rightarrow & {}^{1}1 \\ - & 40 & \rightarrow & 6 \\ \hline & 20 & \rightarrow & 5 \\ \hline \end{array} \; = \; 25$

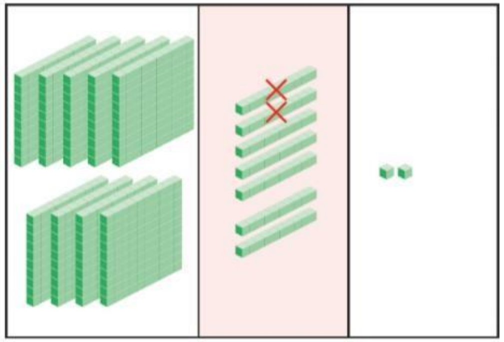
Without renaming:

975 – 723 =

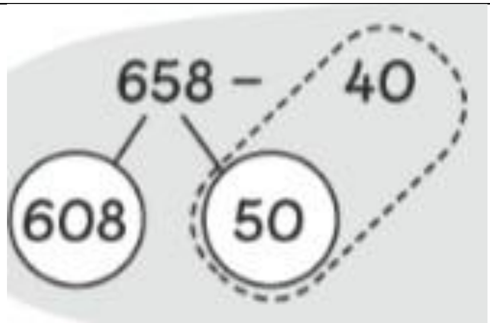
Subtract the Ones first:



Subtract the Tens:



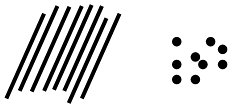
Subtract the Hundreds:



50 – 40 = 10

608 + 10 = 618

Jottings of Base Ten can be used:



	80	→	9	
-	50	→	7	
	30	→	2	= 32

Bar models can be helpful to see the difference:

47 – 24 = 23

	40	+	7
–	20	+	4
	20	+	3

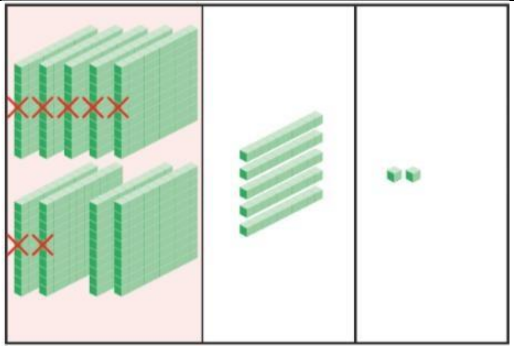
Without renaming:

975 – 723 =

Subtract the Ones first:

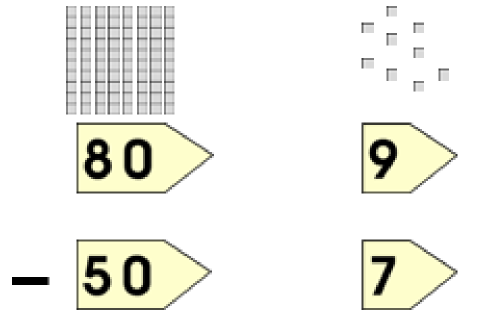
	h	t	o
	9	7	5
–	7	2	3
			2

Subtract the Tens:

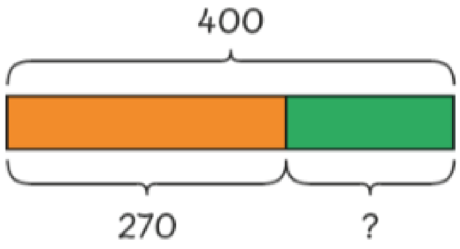


Arrow cards can be used to support and can bridge to the expanded method:

STEP 1:



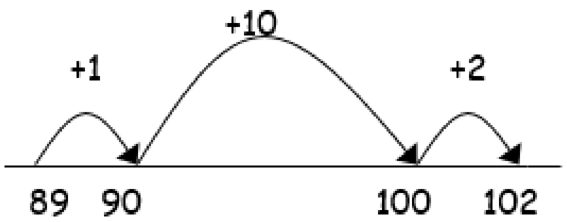
STEP 2:

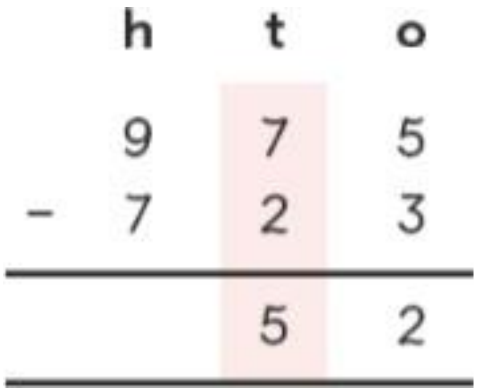


Number lines:

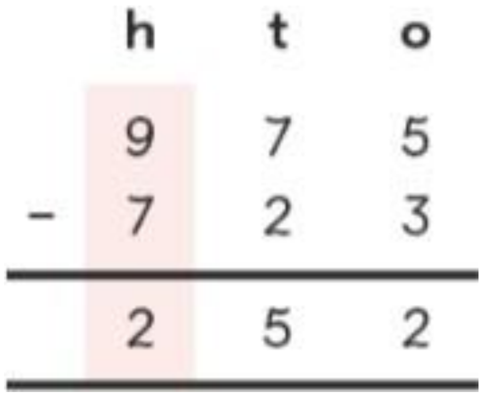
If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, children should be encouraged to recognise that it is more efficient to find the difference by counting up. A number line is useful for this:

102 – 89 = 13



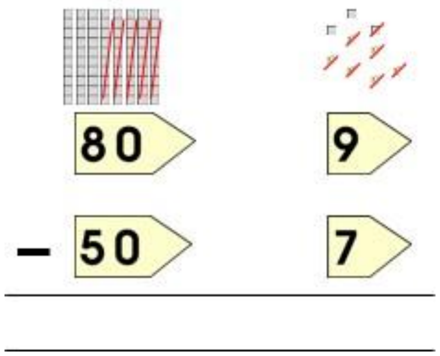


Subtract the Hundreds:

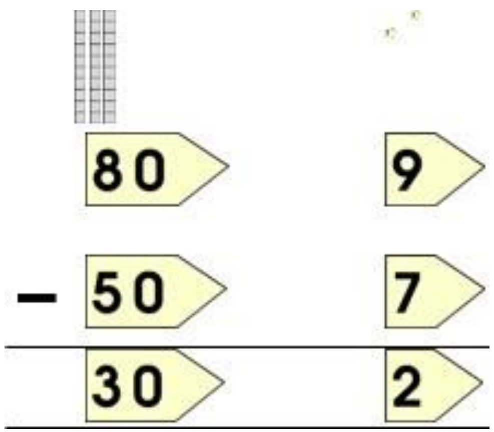


Renaming:

831 – 26 =



STEP 3:

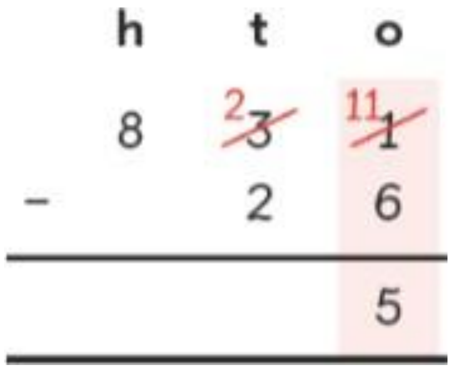


Renaming:

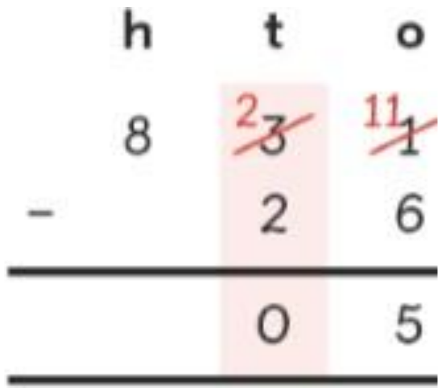
831 – 26 = 805

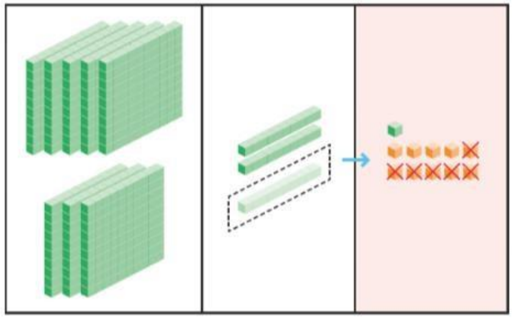
Exchange one Ten from the Hundreds to the Ones column

Exchange one Ten from the Hundreds to the Ones column

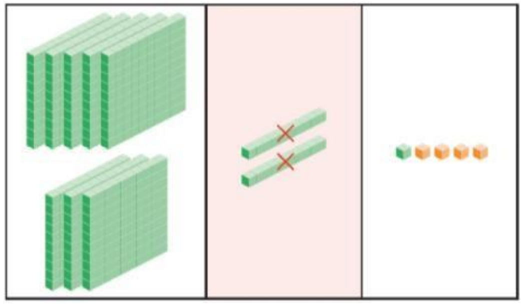


Subtract the Tens:

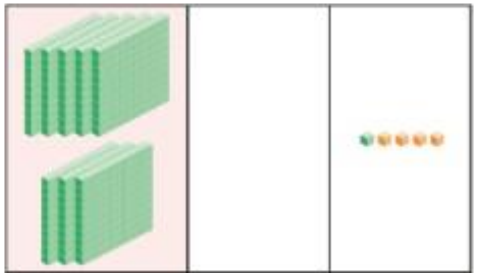




Subtract the Tens:



Subtract the Hundreds:



Subtract the Hundreds:

	h	t	o
	8	~~3~~ 2	~~1~~ 11
−		2	6
	8	0	5

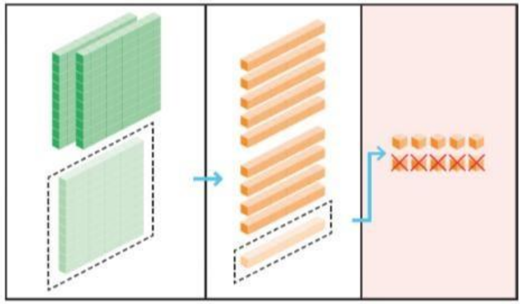
Dealing with the zeros!

300 – 125 =

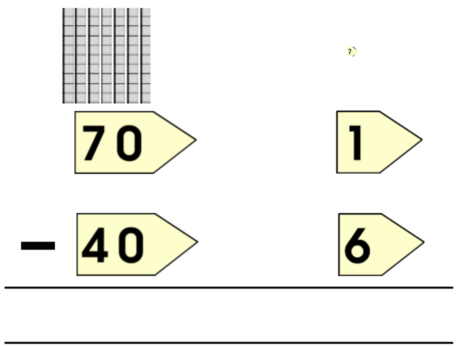
	h	t	o
	~~3~~ 2	~~0~~ ~~10~~ 9	~~0~~ 10
−	1	2	5
			5

300 – 125 =

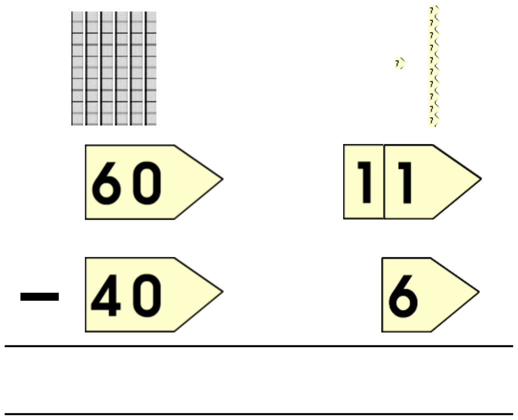
Regroup one Hundred into Tens (x 10) and then regroup one Ten into Ones (x10).
Subtract the Ones



With arrow cards:
STEP 1



STEP 2
Exchange from the Tens:

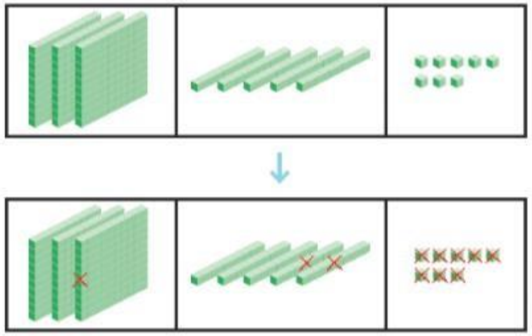
		

YEAR 4 - SUBTRACTION

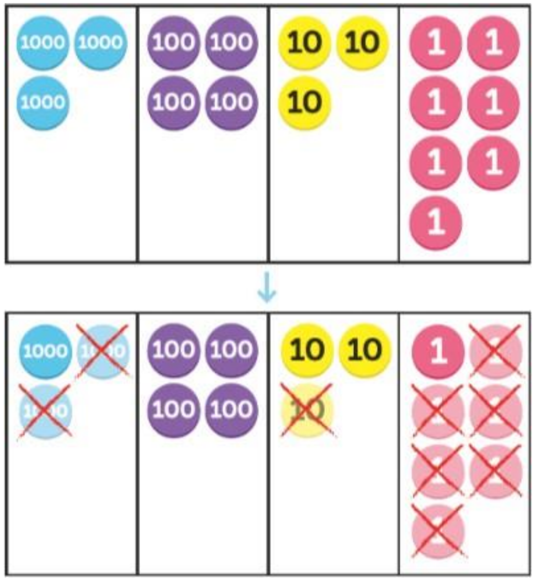
End of Year Objective:

Subtract numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar subtraction where appropriate.

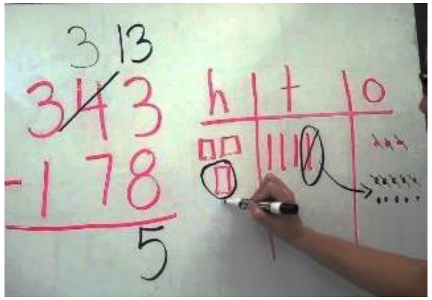
CONCRETE	PICTORIAL	ABSTRACT
Reinforce column method by using concrete materials first, including for renaming. 358 – 128 =	Children use pictorial representations including drawings and images of physical apparatus, as well as the bar model, part whole model and number lines.	By the end of Y4, children should be using the written method confidently and with understanding. They will also be subtracting: - numbers with different numbers of digits, understanding the place value



3437 – 2016 =

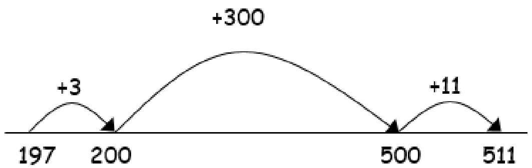


With renaming:



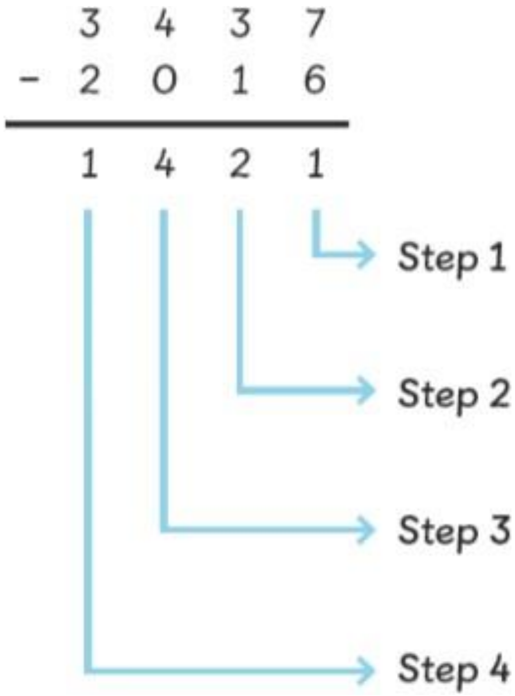
If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, children should be encouraged to recognise that it is more efficient to find the difference by counting up.

511 – 197= 314



- decimals with one decimal place, knowing that the decimal points line up under one another.

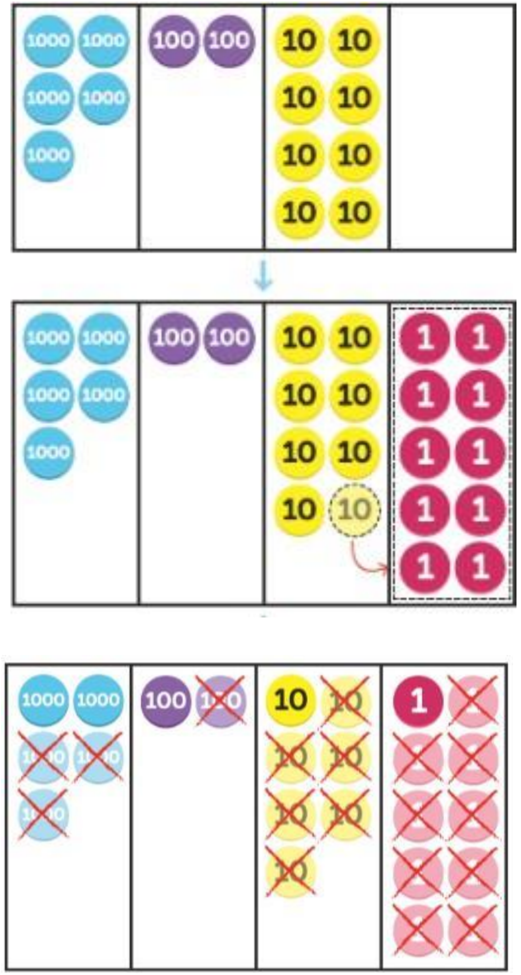
Without renaming:

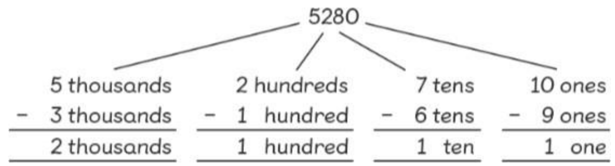


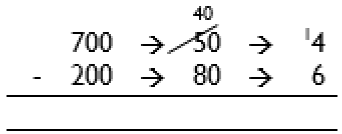
With renaming:

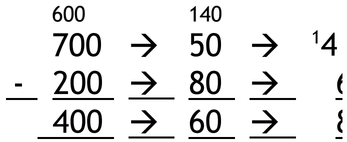
5280 – 3169 = 2111

5280 – 3169 = 2111

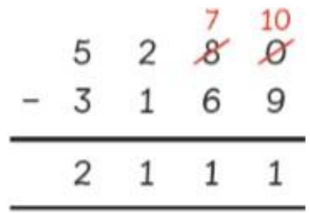


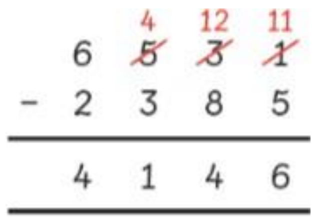






= 468

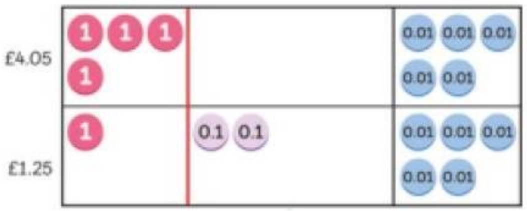


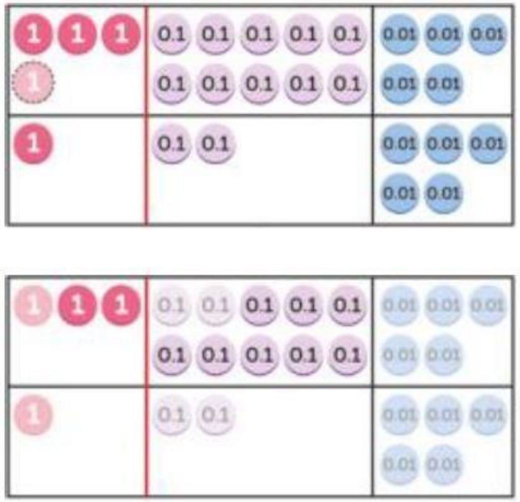
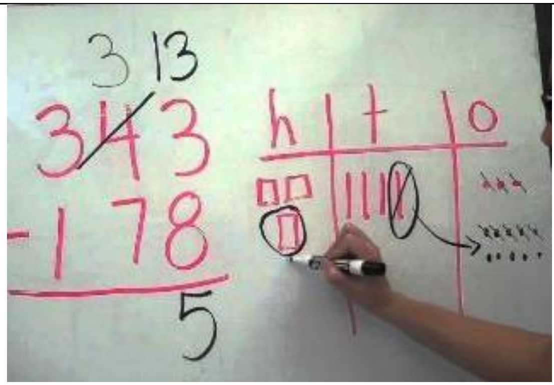


YEAR 5 - SUBTRACTION

End of Year Objective:

Subtract whole numbers with more than 4 digits *and decimals with two decimal places*, including formal written methods (columnar subtraction).

CONCRETE	PICTORIAL	ABSTRACT
At this stage most children working at the expected standard will not need to explore concrete resources for the subtraction of whole numbers as this will have been secured in previous year groups. However, they should be given, planned purposeful opportunities to use counters and Numicon to explore addition of decimals. Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits. £4.05 - £1.25 =  	Children can draw visual representations of place value counters to support the transition from concrete to abstract. As in previous year groups, children can record their drawings of Base 10 using squares for the 100s block, slanted lines for the rods and dots for the ones blocks. 	Children should continue to use the decomposition method to solve calculations such as: $\begin{array}{rrrrr} & ^{6} & & ^{6} & \\ & \cancel{7} & ^{1}0 & \cancel{7} & ^{1}2 \\ - & 3 & 2 & 2 & 6 \\ \hline & 3 & 8 & 4 & 6 \\ \hline \end{array}$ $\begin{array}{rrrrr} & ^{2} & & ^{13} & \\ & \cancel{3} & . & \cancel{4} & ^{1}2 \\ - & 1 & . & 7 & 6 \\ \hline & 1 & . & 6 & 6 \\ \hline \end{array}$ They will also be subtracting:

Exchange from the Ones: 	 	numbers with different numbers of digits, understanding the place value *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.* amounts of money and measures, including those where they have to initially convert from one unit to another

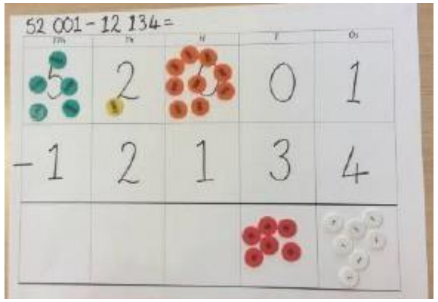
YEAR 6 - SUBTRACTION

End of Year Objective:

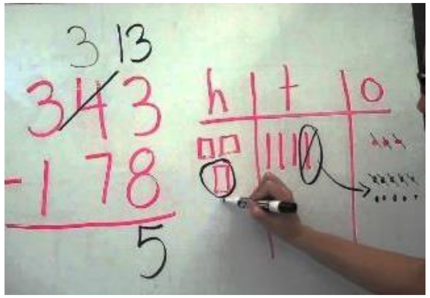
Subtract whole numbers and decimals using formal written methods (columnar subtraction).

CONCRETE	PICTORIAL	ABSTRACT
At this stage most children working at the expected standard will not need to explore concrete resources for the	Children can draw visual representations of place value counters to support the transition from concrete to abstract.	Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits.

subtraction of whole numbers as this will have been secured in previous year groups.
However, they should be given, planned purposeful opportunities to use counters and Numicon to explore addition of decimals.
Children should extend the decomposition method and use it to subtract whole numbers
and decimals with any number of digits.

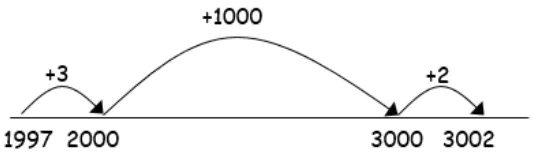


As in previous year groups, children can record their drawings of Base 10 using squares for the 100s block, slanted lines for the rods and dots for the ones blocks.



If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, children should be encouraged to recognise that it is more efficient to find the difference by counting up.

3002 - 1997 = 1005



.

$$\begin{array}{rcccc} & \overset{5}{\cancel{6}} & \overset{13}{\cancel{4}} & {}^{1}3 & 2 \\ - & 4 & 6 & 8 & 1 \\ \hline & 1 & 7 & 5 & 1 \\ \hline \end{array}$$

$$\begin{array}{rccccccc} & \overset{3}{4} & {}^{1}1 & \overset{6}{7} & . & \overset{11}{2} & {}^{1}0 \\ - & & 3 & 4 & . & 7 & 1 \\ \hline & 3 & 8 & 2 & . & 4 & 9 \\ \hline \end{array}$$

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting:

numbers with different
numbers of digits, understanding the place value

decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.

		amounts of money and measures, including those where they have to initially convert from one unit to another Children should: be able to subtract numbers with different numbers of digits be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places know that decimal points should line up under each other.