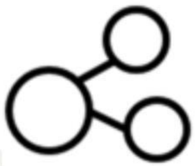
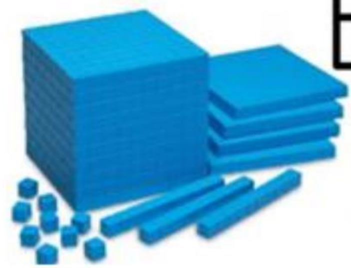
Calculation Policy

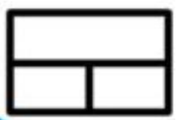
DIVISION

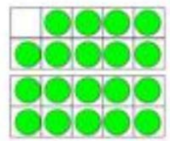
Including Models and Images

October 2022









"The richest concept images will allow children to make the most effective numerical connections, enabling them to communicate mathematically."

Introduction

This policy exemplifies a recommended progression through both mental and written calculations for the four operations and has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of Mathematics. There is a strong focus on the use of models and images to support children's concept image of number and their understanding of how this relates to methods of calculation. It is also designed to give pupils a consistent and smooth progression of learning in calculations across the school, taking into account Maths No Problem! : a Singaporean teaching style in Maths.

The Calculation Policy shows methods that pupils will be taught within their respective year group. It is shown in teaching order. Children should be confident in choosing and using a strategy that they know will get them to the correct answer as efficiently as possible; pupils are free to choose their preferred method to solve calculations. Although our policy is set out based on The National Curriculum year group expectations, children work through the calculation policy systematically. Some children may therefore be working below year group expectation and should be taught the method appropriate for their individual stage in learning. This process should not be rushed; children should be moved on when they are ready.
"Children develop/learn in different ways and at different rates" – EYFS Principles.

Up to Year 3 the main emphasis should be on children working practically and mentally and recording through jottings. Once written methods are introduced, using practical images to support and develop mathematical understanding, mental skills must be kept sharp by continuing to develop and apply them with appropriate examples.

Should children be taught one standard method for each operation?
Children should work through the school's agreed progression in methods in order that they know and understand a compact standard method for each numerical operation by the end of Year 6.

How can children's readiness for written calculations be judged?
Judgements will need to be made as to whether pupils possess sufficient of these skills to progress. Different prerequisite skills are needed for each operation.

A short list of criteria for readiness for written methods of addition and subtraction would include:

- Do children know addition facts to 20?
- Do they understand place value and can they partition numbers into hundreds, tens and units?
- Do they use and apply commutative and associative laws of addition?
- Can they add at least three 1-digit numbers mentally?
- Can they add and subtract any pair of 2-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Corresponding criteria to indicate readiness to learn written methods for multiplication and division are:

- Do the children know their 2,3,4,5 and 10 times tables and corresponding division facts?
- Do they know the result of multiplying by 0 or 1?
- Do they understand 0 as a place holder?
- Can they multiply 2 and 3 digits mentally by 10 and 100?
- Can they use their knowledge of all the multiplication tables to approximate?
- Can they find products using multiples of 10?
- Do they use the commutative and associative laws of multiplication?
- Can they halve and double 2-digit numbers mentally?
- Can they use multiplication facts to derive mentally, other multiplication facts they don't know?
- Can they explain their mental strategies orally and record them using informal jottings?

Concrete, Pictorial, Abstract:

A key principle behind the Singapore Maths textbooks and Maths Mastery is based on the concrete, visual and abstract approach. Pupils are first introduced to an idea or skill by acting it out with real objects (a hands-on approach). Pupils then are moved onto the visual stage, where pupils are encouraged to relate the concrete understanding to pictorial representations. The final abstract stage is a change for pupils to represent problems by using mathematical notation.

Whilst this calculation policy aims to show the CPA approach to the different calculations, it is not always noted further up the year groups. However, it is expected that the CPA approach is used continuously in all new learning and calculations even when not noted.

Reasoning and Problem Solving:

Once children are fluent in the calculation strategy for their year group, we deepen and embed understanding through providing children with a range of reasoning and problem solving skills that allow the children to show their full understanding in a range of different context.

Monitoring of Written Calculations

It is important that procedures are in place to ensure that all staff are aware of the progression through calculations, and that children are being taught appropriate methods for their age and ability, which are in line with the agreed policy. This may include book sampling, reflective enquiries, monitoring of planning, learning walks and pupil interviews.

Progression for Division ÷

Key Vocabulary: share, divide, remainder, divisor, factor, quotient, dividend,

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction
- sharing into equal amounts

They also need to understand and work with certain principles, i.e. that it is:

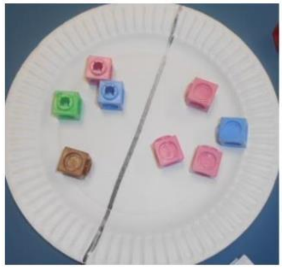
- the inverse of multiplication
- **not** commutative i.e. 15 ÷3 is not the same as 3 ÷ 15
- **not** associative i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷ 2

EARLY LEARNING GOAL:

Children solve problems, including halving and sharing.

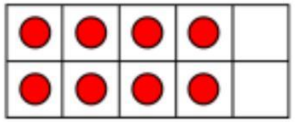
In **EYFS** pupils should be developing their concept of the number system through the use of concrete materials and pictorial representations. They should experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc. It is vital to develop a deep number sense so Number Talk is really important!

Pupils should have many practical experiences of sharing objects in role play e.g. sharing between 2 people, or finding ½ of a group of objects.

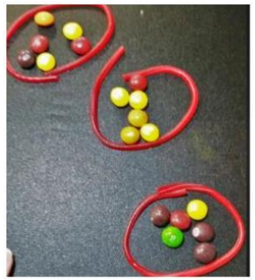


Children find halves of objects in shapes and practical situations, for example cutting fruits in half.

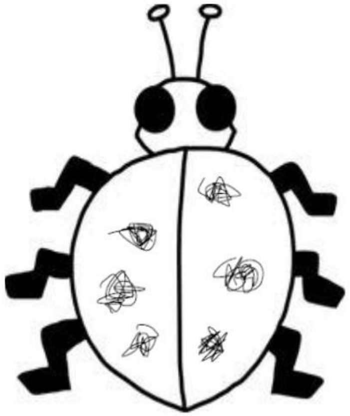
Use a range of concrete materials to show a number and then share them equally. Children
may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays:



Can they share the cakes so that each teddy has the same?

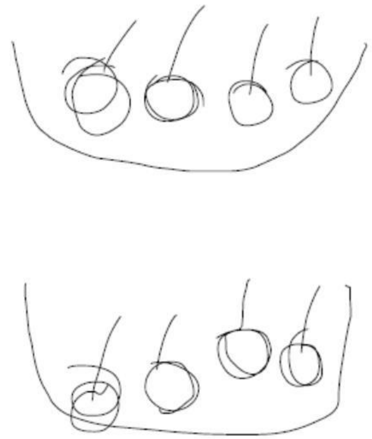


Then move onto pictorial representations and may develop ways of recording calculations using pictures, etc:



A child's jotting showing halving six spots between two sides of a ladybird.

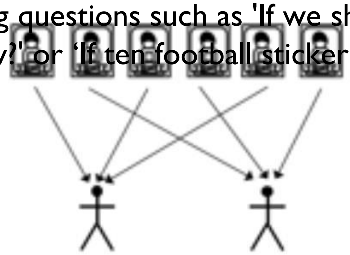
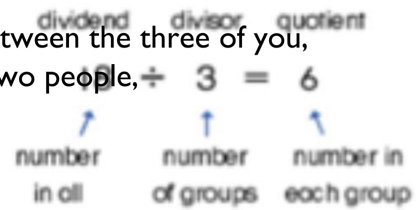
A child's jotting showing how they shared the apples at snack time between two groups.

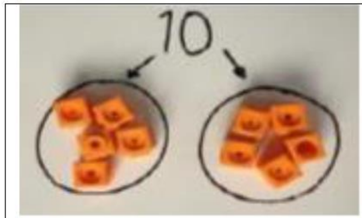


YEAR 1 – DIVISION ÷

End of Year Objective:

Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

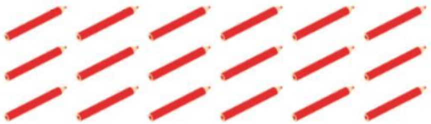
CONCRETE	PICTORIAL	ABSTRACT
They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If ten football stickers are shared between two people, how many do they each get?'	Sharing 6 cards between two people: 	



They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.

Sharing groups equally:

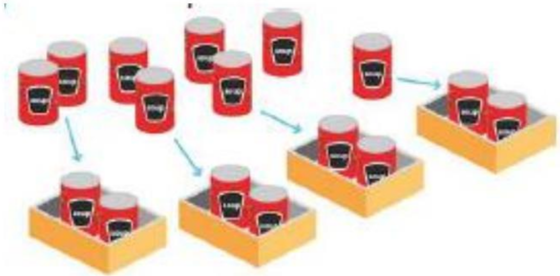
Objects can be sorted into equal groups e.g. boxes of 6 pencils



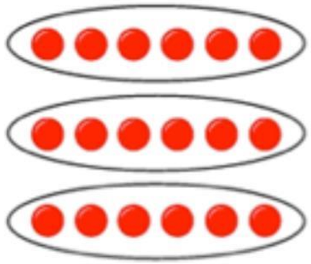
Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '… left over'.

Also worded: Two people are playing a card game. **Each** person gets 6 cards.

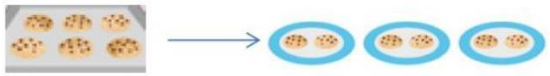
Sally has 8 cans of soup. She packs them equally in 4 boxes. There are $\boxed{}$ cans in each box.



There are 18 counters. Circle groups of 6.



Ravi has baked 6 cookies and gives 2 to each of his children. How many children has he got?



In a real life word problem:

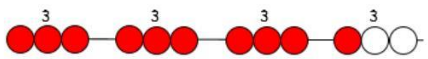
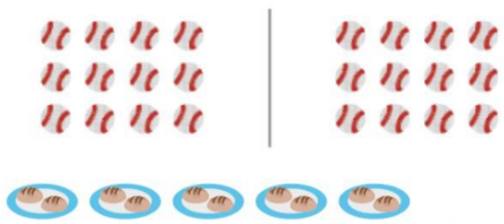
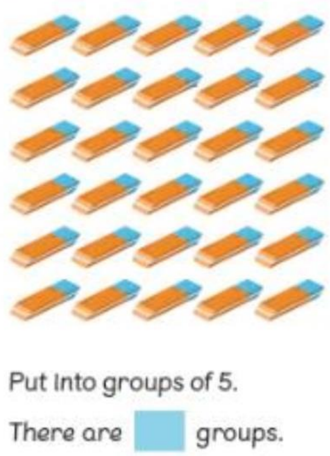
5 children share 10 sweets equally

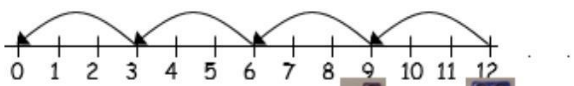
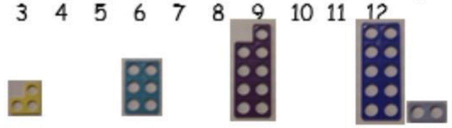
12 shared between 3 is 4

YEAR 2 – DIVISION

End of Year Objective:

Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs.

CONCRETE	PICTORIAL	ABSTRACT
Build on Year 1 by consolidating grouping equally. Use a range of resources to show division. Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g. 12 ÷ 3 =  Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'  Work on each times table in order (2, 5 then 10). Use idea of grouping before	Dividing by 2:  Dividing by 5:  Put into groups of 10:	Associate to the law of commutativity to show link between multiplication and division: $6 \times 10 = 60$ $60 \div 10 = 6$ $5 \times 2 = 10$ —— $10 \div 2 = 5$ $2 \times 5 = 10$ —— $10 \div 5 = 2$ Using symbols to stand for unknown numbers to complete equations using inverse operations: $\square \times 10 = 50$

show division and link to the abstract calculation with the ÷ sign. They should also continue to develop their knowledge of division with remainders, e.g. 13 ÷ 4 =  13 ÷ 4 = 3 remainder 1 Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly.	 60 sweets shared between 10 people is 6  **Repeated subtraction using a numberline or bead bar:** 12 ÷ 3 = 4  	$\square \div 2 = 4$ $20 \div \square = 4$ $\square \div \square = 4$ Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation 13 ÷ 4, the answer is 3 remainder1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below: I have £13. Books are £4 each. How many can I buy? Answer: 3 (the remaining £1 is not enough to buy another book) Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed? Answer: 4 (the remaining 1 apple still needs to be placed into a box)

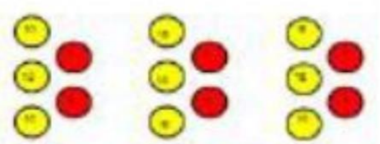
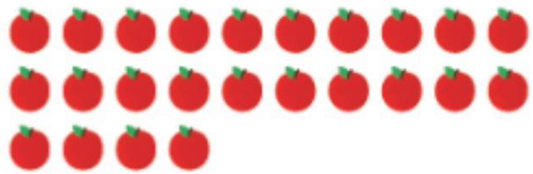
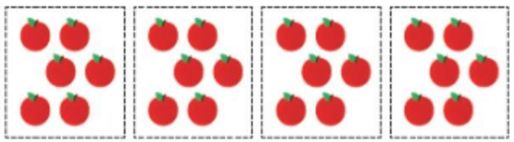
YEAR 3 – DIVISION

End of Year Objective:

Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.*

***Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4**

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

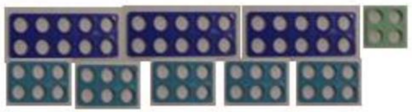
CONCRETE	PICTORIAL	ABSTRACT
Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10) e.g. With place value counters: 96 ÷ 3 = 32 	Put 24 apples into 4 groups:   12 ÷ 3 = 4	63 ÷ 3 = 21 6 Tens ÷ 3 Tens = 2 Tens 3 Ones ÷ 3 Ones = 1 One 88 ÷ 8 = 11 8 Tens ÷ 8 Tens = 1 Ten 8 Ones ÷ 8 Ones = 1 One **Formal method of chunking:** Encourage children to first generate key facts for the divisor, looking at relationships to calculate the answer:

15 ÷ 3 = 5
15 ÷ 5 = 3

With remainders:

43 ÷ 8 = 5 remainder 3

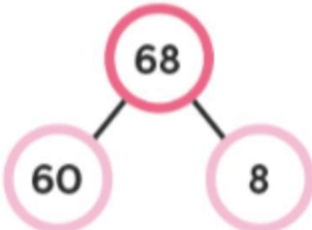
Children will begin practically to think about how many 6s fit into 34:



And when dividing into groups, what is left over:

68 ÷ 2 = 34

Use part whole to partition:



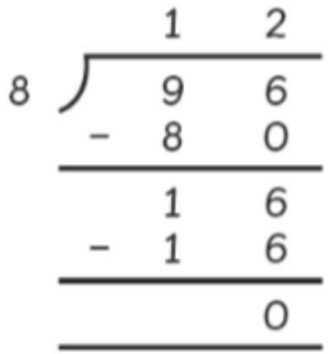
First divide the Tens:
Then divide the Ones:
Add the results
30 + 4= 34

With regrouping:

Children can use their multiple knowledge to partition to divide equally:

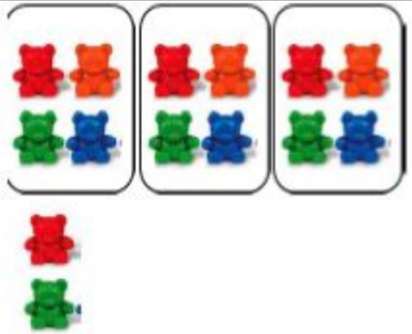
Key facts box

1x	8
2x	16
5x	40
10x	80



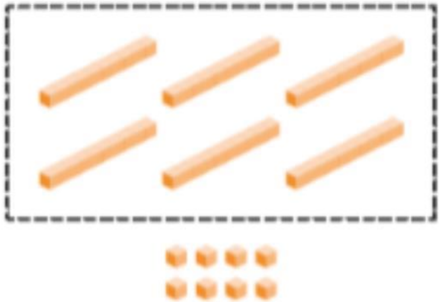
Stage 1 – repeatedly subtracting individual groups of the divisor
Stage 2 – subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)

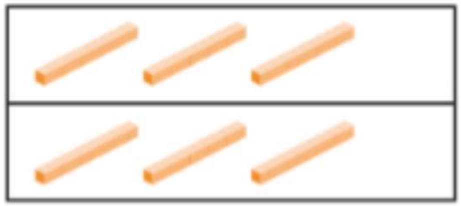
After each group has been subtracted (chunked), children should consider how many are left.



68 ÷ 2 = 34

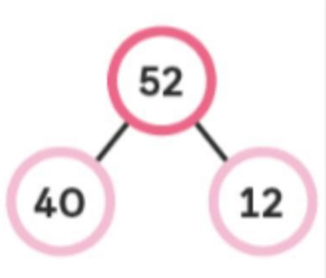
First divide the Tens:





Then divide the Ones:

52 ÷ 4 =

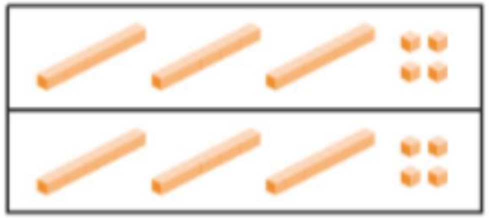


Now divide the Tens
Then divide the Ones

Children can make a family of multiplication and division equations:

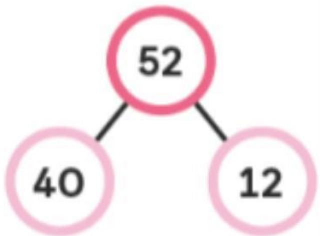
$20 \div 4 = 5$
$5 \times 4 = 20$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

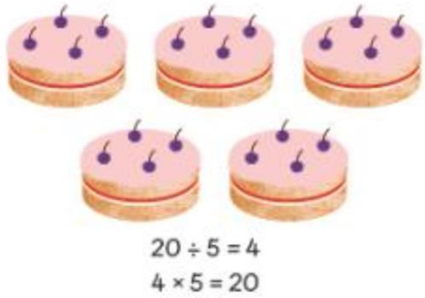


Add the results
30 + 4= 34

With regrouping:
Children can use their multiple knowledge to partition to divide equally:
52 ÷ 4 =



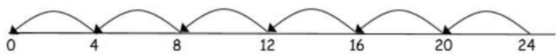
Now divide the Tens:



Children will continue to use grouping (repeated subtraction) to represent their calculations, answering questions such as:
24 ÷ 4 = 6

Children will use an empty number line to support their calculation.

24 ÷ 4 = 6



After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

Bar model can also be used for problem-solving:

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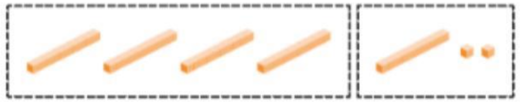
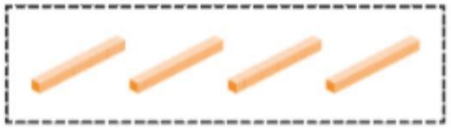

40 ÷ 4 =

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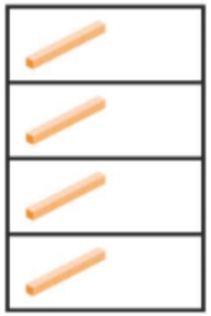
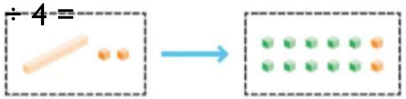
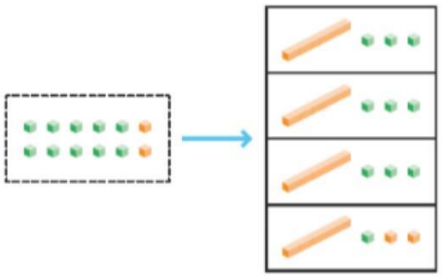

Then regroup the one Ten into ten Ones. 12 ÷ 4 =


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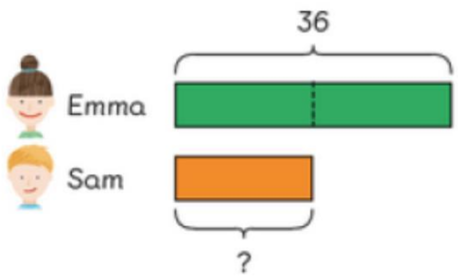
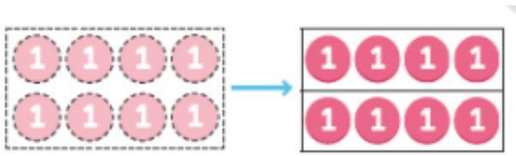
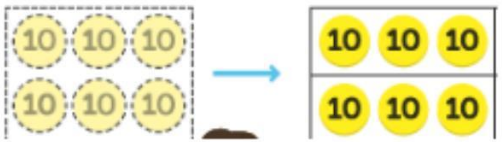
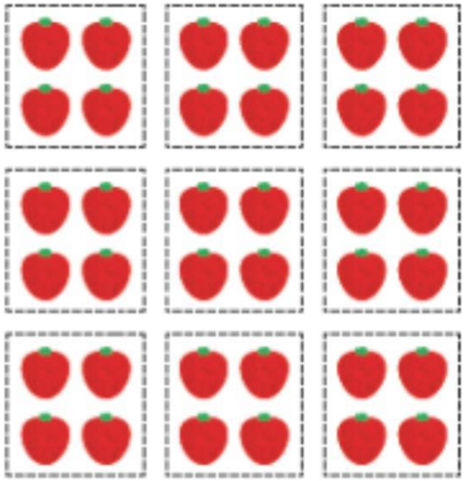


36 ÷ 2 = 18
Sam has 18 beads.
</td>
<td></td>
</tr>
</table>

YEAR 4 – DIVISION ÷

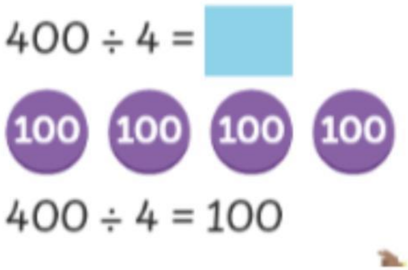
End of Year Objective:

Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

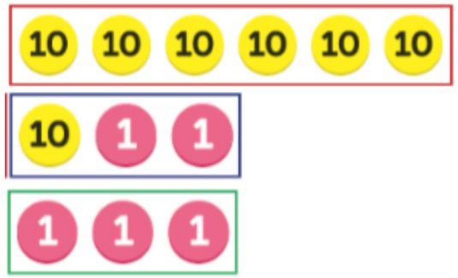
CONCRETE	PICTORIAL	ABSTRACT
Children use practical resources to partition and share into groups. 68 ÷ 2 = 34   Total up in one group: 3 Tens and 4 Ones	Children continue to share into equal groups: 36 ÷ 9 = 4  Bar model:	Children can continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, until they feel confident to move onto short division. 3 4 2) 6 8 − 6 8 − 8 0 14 4) 56 - 40 10x 16 - 16 4x 0 Answer: 14 Children should write their answer above the calculation to make it easy for them and the teacher to distinguish. When developing their understanding of short division, children should utilise a 'key facts' box, as

408 ÷ 4 = 34

8 Ones divided by 4 is 2 Ones

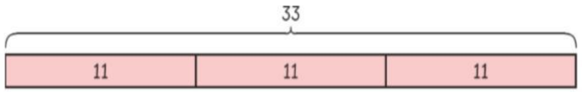


75 ÷ 6 = 9

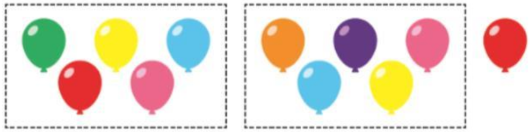


6 Tens divided by 6 is 1 Ten= 10

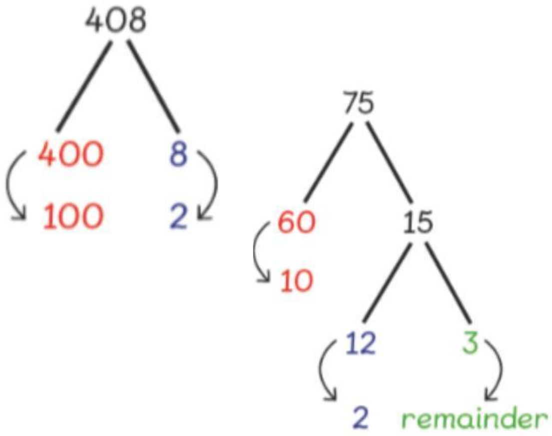
Exchange one Ten for ten Ones and then divide 15 Ones by 6=
2 remainder 3



11 ÷ 2 = 5 remainder 1



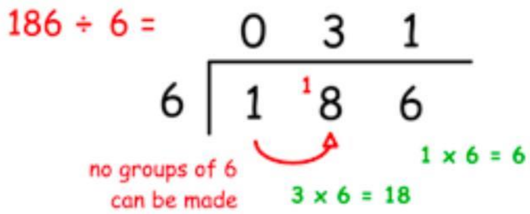
39 ÷ 4 = 9 remainder 3



shown below. This enables an efficient recall of tables facts and will help them. Any remainders should be shown as integers.

Key facts box

1x6=6	5x6= 30
2x6=12	6x6=36
3x6=18	7x6=42
4x6=24	8x6=48
	9x6= 54

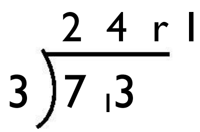


Questions to support calculation should be:

What do I multiply 6 by to get a multiple near 18 but not over it? Answer: 3

What do I multiply 6 by to get a multiple near 6 but not over it? Answer: 1

With a remainder:



Key facts box

1x3=3	5x3=15
2x3=6	6x3=18
3x3=9	7x3=21
4x3=12	8x3=24
	9x3= 27

$$\begin{array}{r} 0\;3\;3 \text{ r } 1 \\ 3\overline{)1\,{}_{1}0\,{}_{1}0} \end{array}$$

By the end of Year 4, children should be able to use the short division method to divide a **three digit** number by a single digit number.

196 ÷ 6

$$\begin{array}{r} 3\;2 \text{ r } 4 \\ 6\overline{)1\,{}_{1}9\,{}_{1}6} \end{array}$$

Key facts box

1x 6=6	6x6=36
2x6=12	7x6=42
3x6=18	8x8=48
4x6=24	9x8=72
5x6=30	

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

YEAR 5 – DIVISION

End of Year Objective:

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

CONCRETE

Children can continue to use place value counters to reinforce the place value of the digits:

With remainders:

PICTORIAL

When chunking, children can use strategies to show equal groups of the multiple:

150 ÷ 6 =

ABSTRACT

Children may continue to use the key facts box for as long as they find it useful. **Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor**. Any remainders should be shown as integers, e.g.

Children should be confident with the short division method.

Firstly with no remainder:

Once the children are confident at dividing a 2-digit number by a 1-digit number and have been given the precious concrete and pictorial experiences most children will not need the concrete and pictorial approach.

The number line can be used to ‘chunk’ off multiples of the divisor:

5048 ÷ 8

Key facts box:

1x 8 = 8	4 x 8 = 32	7 x 8= 56
2 x 8 = 16	5 x 8= 40	8 x 8 =64
3 x 8= 24	6 x 8= 48	9 x 8 = 72

$$\begin{array}{r} 0\;6\;3\;1 \\ 8\;\overline{)\,5\;{}_{5}0\;{}^{2}4\;8} \end{array}$$

<u>Questions to support calculation should be:</u>

What do I multiply 8 by to get a multiple near 50 but not over it? Answer: 6

What do I multiply 8 by to get a multiple near 24 but not over it? Answer: 3

What do I multiply 8 by to get a multiple of 8 but not over it? Answer: 1

4894 ÷ 4

Key facts box:

1x 4 = 4	4 x 4 = 16	7 x 4= 28
2 x 4 = 8	5 x 4= 20	8 x 4 =32
3 x 4= 12	6 x 4= 24	9 x 4 = 36

1 2 2 7

4 4 8 9 ₁4

With a remainder:

6613 ÷ 5

Key facts box:

1x 5 = 5	4 x 5 = 20	7 x 5= 35
2 x 5 = 10	5 x 5= 25	8 x 5 =40
3 x 5= 15	6 x 5= 30	9 x 5 = 45

1 3 2 2 r 3

5 6 6 1 3

Questions to support calculation should be:

What do I multiply 5 by to get a multiple near 6 but not over it? Answer: 1

What do I multiply 5 by to get a multiple near 16 but not over it? Answer: 3

What do I multiply 5 by to get a multiple of 11 but not over it? Answer: 2

What do I multiply 5 by to get a multiple of 13 but not over it? Answer: 2 r 3

This also can reinforce understanding about common fractions and decimals:

Then with a remainder:

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Chidren need to be able to express the remainder as a fraction ‘of the divisor’ as well as a decimal:

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

YEAR 6 – DIVISION

End of Year Objective:

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

CONCRETE

For simple fraction and decimal equivalents, this could also be demonstrated using a simple calculation such as 13 ÷ 4 to show the remainder initially as a fraction.

Using practical equipment, children can see that for 13 ÷ 4, the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is 1/4. To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

PICTORIAL

The number line can be used to 'chunk' off multiples of the divisor:

ABSTRACT

To develop the chunking method further, children should be taught the formal method of long multiplication: it should be extended to include dividing a four-digit number by a two-digit number, e.g.

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Answer: 122

or 122 remainder 1
or 122.33*

Dividing with decimals:

In addition, children should also be able to use the long multiplication method and solve calculations interpreting the remainder as a decimal up to two decimal places, e.g.

Answer: 158.75

362 ÷ 17

Key facts box:

1x 17 = 17	6 x 17= 102
2 x 17 =34	7 x 17= 119
3 x 17= 51	8 x 17= 136
4 x 17= 68	9 x 17= 153
5 x 17= 85	

3574 ÷ 8

Key facts box:

1x 8 = 8	6 x 8= 48
2 x 8 = 16	7 x 8= 56
3 x 8= 24	8 x 8= 64
4 x 8= 32	9 x 8= 72
5 x 8= 40	10 x 8= 80

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

and reduce other equivalent fractions to their lowest terms.