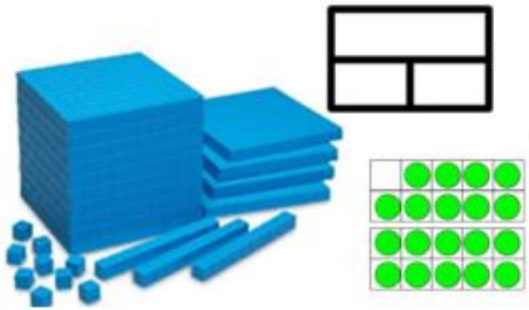
Calculation Policy

MULTIPLICATION

Including Models and Images

October 2022



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"The richest concep enabling tllow children to make mathematical hem children to make the most effective numerical connections, y."

Introduction

This policy exemplifies a recommended progression through both mental and written calculations for the four operations and has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of Mathematics. There is a strong focus on the use of models and images to support children's concept image of number and their understanding of how this relates to methods of calculation. It is also designed to give pupils a consistent and smooth progression of learning in calculations across the school, taking into account Maths No Problem! : a Singaporean teaching style in Maths.

The Calculation Policy shows methods that pupils will be taught within their respective year group. It is shown in teaching order. Children should be confident in choosing and using a strategy that they know will get them to the correct answer as efficiently as possible; pupils are free to choose their preferred method to solve calculations. Although our policy is set out based on The National Curriculum year group expectations, children work through the calculation policy systematically. Some children may therefore be working below year group expectation and should be taught the method appropriate for their individual stage in learning. This process should not be rushed; children should be moved on when they are ready.
"Children develop/learn in different ways and at different rates" – EYFS Principles.

Up to Year 3 the main emphasis should be on children working practically and mentally and recording through jottings. Once written methods are introduced, using practical images to support and develop mathematical understanding, mental skills must be kept sharp by continuing to develop and apply them with appropriate examples.

Should children be taught one standard method for each operation?

Children should work through the school's agreed progression in methods in order that they know and understand a compact standard method for each numerical operation by the end of Year 6.

How can children's readiness for written calculations be judged?

Judgements will need to be made as to whether pupils possess sufficient of these skills to progress. Different prerequisite skills are needed for each operation.

A short list of criteria for readiness for written methods of addition and subtraction would include:

- Do children know addition facts to 20?
- Do they understand place value and can they partition numbers into hundreds, tens and units?
- Do they use and apply commutative and associative laws of addition?
- Can they add at least three 1-digit numbers mentally?
- Can they add and subtract any pair of 2-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Corresponding criteria to indicate readiness to learn written methods for multiplication and division are:

- Do the children know their 2,3,4,5 and 10 times tables and corresponding division facts?
- Do they know the result of multiplying by 0 or 1?
- Do they understand 0 as a place holder?
- Can they multiply 2 and 3 digits mentally by 10 and 100?
- Can they use their knowledge of all the multiplication tables to approximate?
- Can they find products using multiples of 10?
- Do they use the commutative and associative laws of multiplication?
- Can they halve and double 2-digit numbers mentally?
- Can they use multiplication facts to derive mentally, other multiplication facts they don't know?
- Can they explain their mental strategies orally and record them using informal jottings?

Concrete, Pictorial, Abstract:

A key principle behind the Singapore Maths textbooks and Maths Mastery is based on the concrete, visual and abstract approach. Pupils are first introduced to an idea or skill by acting it out with real objects (a hands-on approach). Pupils then are moved onto the visual stage, where pupils are encouraged to relate the concrete understanding to pictorial representations. The final abstract stage is a change for pupils to represent problems by using mathematical notation.

Whilst this calculation policy aims to show the CPA approach to the different calculations, it is not always noted further up the year groups. However, it is expected that the CPA approach is used continuously in all new learning and calculations even when not noted.

Reasoning and Problem Solving:

Once children are fluent in the calculation strategy for their year group, we deepen and embed understanding through providing children with a range of reasoning and problem solving skills that allow the children to show their full understanding in a range of different context.

Monitoring of Written Calculations

It is important that procedures are in place to ensure that all staff are aware of the progression through calculations, and that children are being taught appropriate methods for their age and ability, which are in line with the agreed policy. This may include book sampling, reflective enquiries, monitoring of planning, learning walks and pupil interviews.

Progression for Multiplication ×

Key Vocabulary: multiply, multiple, groups of, times, lots of, repeated addition, product

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of division
- commutative i.e. 5 x 3 is the same as 3 x 5
- associative i.e. 2 x 3 x 5 is the same as 2 x (3 x 5)

EARLY LEARNING GOAL:

Children solve problems, including doubling.

In **EYFS** pupils should be developing their concept of the number system through the use of concrete materials and pictorial representations. They should experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc. It is vital to develop a deep number sense so Number Talk is really important!

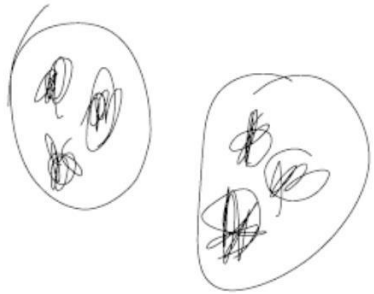
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

e.g. There are 3 apples on each plate. How many apples are there altogether?

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.

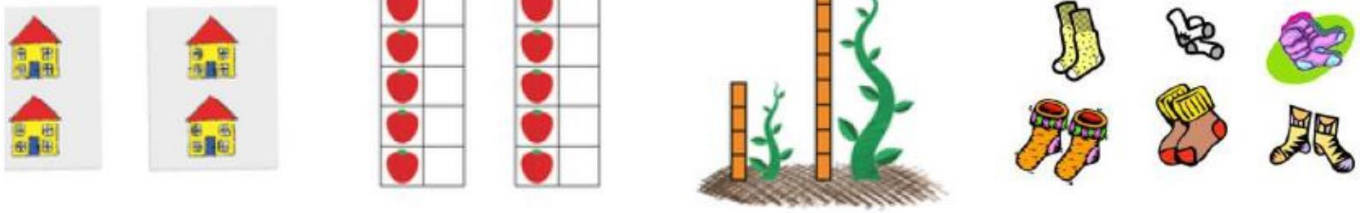
They may develop ways of recording calculations using pictures, etc.

A child's jotting showing the fingers on each hand as a double.



A child's jotting showing double three as three cookies on each plate.

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.They will use a range of concrete materials to show a number and then repeat the number to show doubling. Then move onto pictorial representations:



Children double numbers to 5 using practical equipment, pictures and symbols.

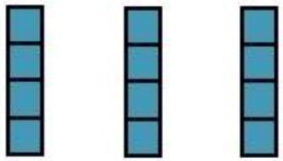
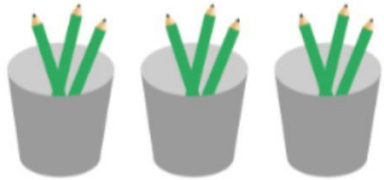
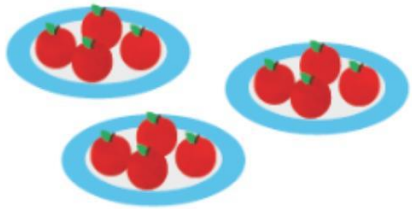
When appropriate, children may begin to put items into groups and count them e.g. pairs of socks.

YEAR 1 - MULTIPLICATION

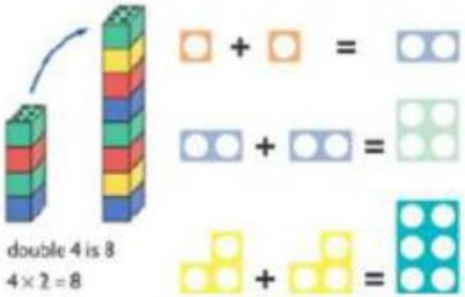
End of Year Objective:

Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

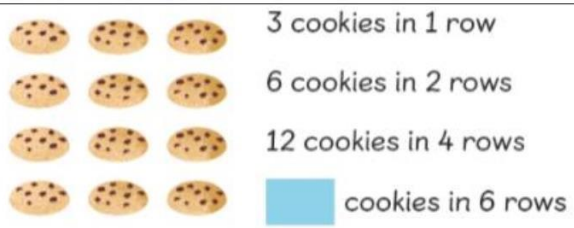
CONCRETE	PICTORIAL	ABSTRACT
Children will continue to solve multiplication problems using practical equipment and jottings. They may use	Children should practise making equal groups first and add them to associate repeated addition with multiplication. They	Children count in multiples of a number aloud: 2, 4, 6, 8, 10

the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?' Children should be introduced to the concept of repeated addition, through a range of different practical resources such as numicon, counters and multilink e.g.  	will move on to use pictorial representations:     Associate grouping to equal rows so children learn to count up in the same number:	5, 10, 15, 20, 15, 4 groups of 2 = 4 groups of 5 = 4 groups of 10= 5 twos are……. 2 nines are ……

Use practical activities using manip-ultives including cubes and Numicon to demonstrate doubling



Associate grouping to equal rows so children learn to count up in the same number:



Drawings

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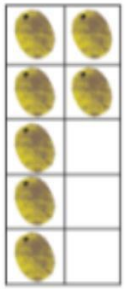
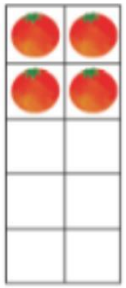
Ten frames can help with doubling:

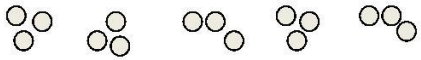
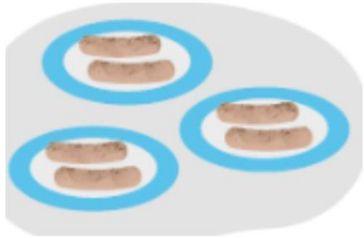
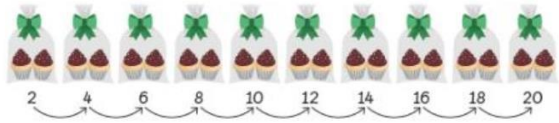




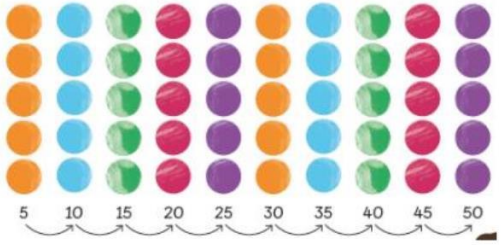
YEAR 2 - MULTIPLICATION ×

End of Year Objective:

Calculate mathematical statements for multiplication *(using repeated addition)* and write them using the multiplication (x) and equals (=) signs.

CONCRETE	PICTORIAL	ABSTRACT
Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g. 3 x 5 can be shown as five groups of three with counters, either grouped in a random pattern, as below  or in a more ordered pattern, with the groups of three indicated by the border outline: 	Children should focus on doubles, fives and tens:   	Children should be able to write different number sentences: 2 + 2 + 2 + 2= 8 4 twos = 8 4 groups of 2 = 8 4 x 2= 8 Explore commutativity:

Children should understand that an array can represent different equations and



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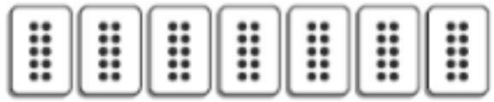
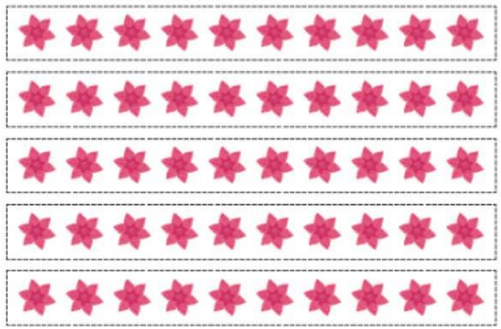
7 x 10 = 70





Children use different representations of arrays to show different calculations and explore commutativity:

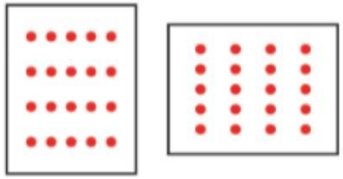
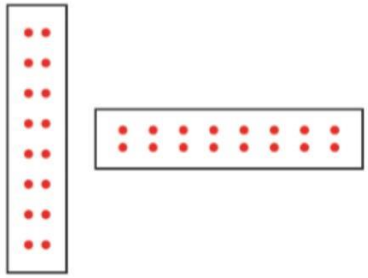
5 x 4 = 20
4 x 5 = 20

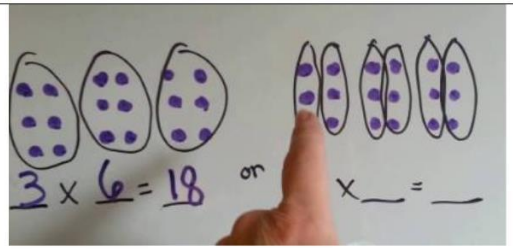
4 × 5 = □

5 × 4 = □

Children apply their skills in word problems:

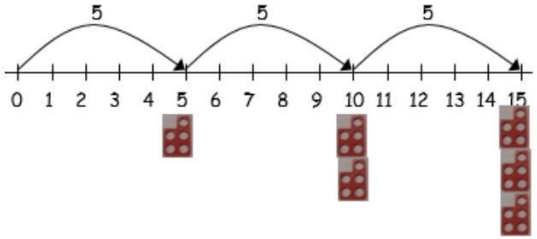
There are 5 toy cars. Each toy car has 4 wheels. How many wheels are there altogether?

because multiplication is commutative, the order of the calculation does not affect the answer: 5 x 2 = 10 2 x 5 = 10	 2 x 8 = 16 8 x 2 = 16  2 x 4 is the same as 4 x 2  Children can draw their own arrays to show commutativity:	

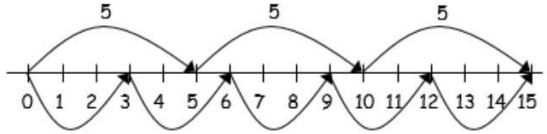


Repeated addition can also be shown easily on a number line:

5 x 3 = 5 + 5 + 5



Also commutativity:



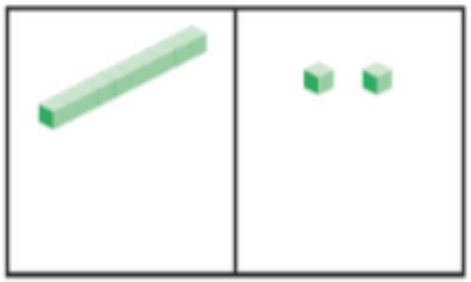
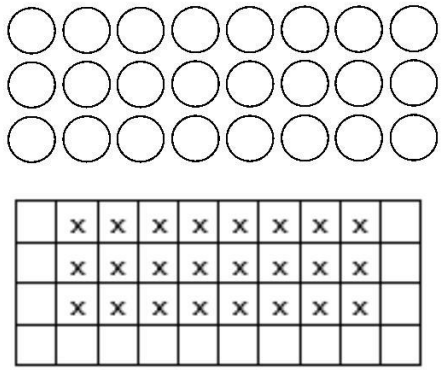
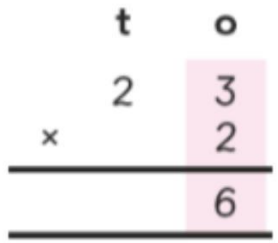
YEAR 3 - MULTIPLICATION ×

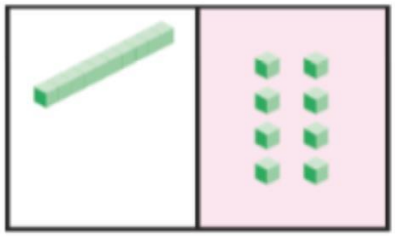
End of Year Objective:

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.*

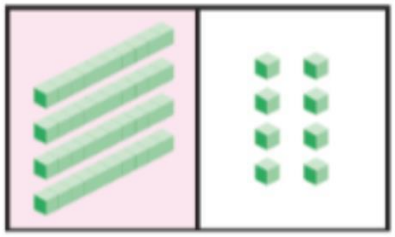
**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

CONCRETE	PICTORIAL	ABSTRACT
Children use practical resources such as Base Ten to partition: 12 x 4 =  Multiply the Ones by 4:	Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g. 	Consolidate repeated addition before moving onto multiplication of 2 digit numbers. -Multiply the ones digit by the single-digit number -Multiply the tens digit by the single-digit number 

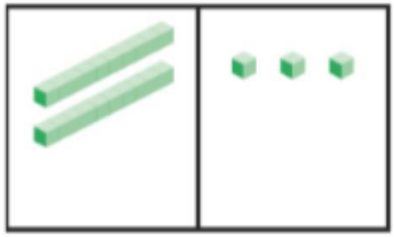


Multiply the Tens by 4:



With regrouping:

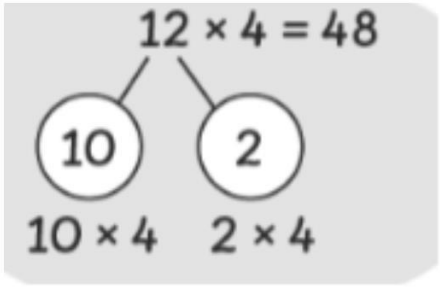
23 x 4 =



Multiply the Ones: we can exchange ten Ones for one Ten

8 x 3 = 8 + 8 + 8 = 24
3 groups of 8
3 x 8 = 24
There are 24 legs in total

Partition using part whole:



Children will continue to use:

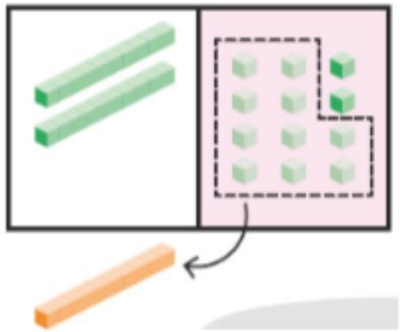
Repeated addition

6 times 4 is
6 + 6 + 6 + 6 = 24
or
4 lots of 6
or 6 x 4

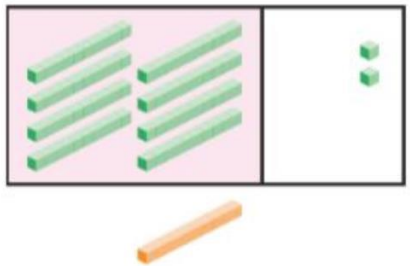
	t	o
	2	3
×		2
		6
	4	0

	t	o
	2	3
×		2
		6
+	4	0
	4	6

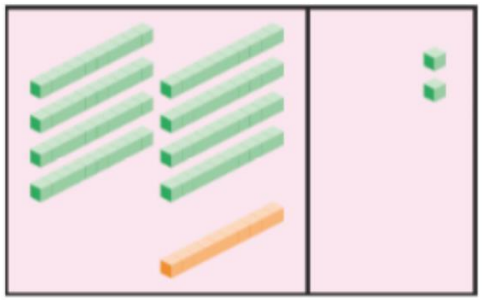
Regrouping (expanded):
23 x 4 =



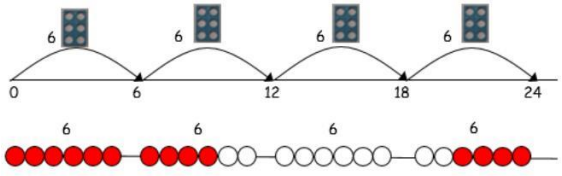
Multiply the Tens: remember e need to add the exchanged Ten



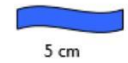
Add them all up:

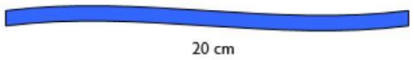


Children should use number lines or bead bars to support their understanding.

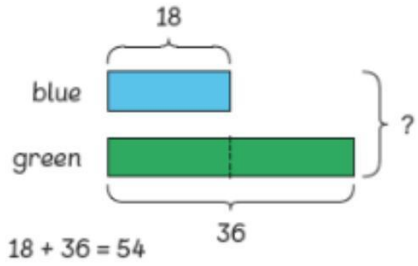


Children develop their understanding about **Scaling:**
e.g. Find a ribbon that is 4 times as long as the blue ribbon





The bar model can help solve word problems:



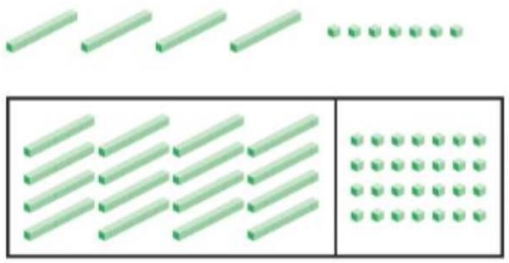
	t	o
	2	3
×		4
	1	2

	t	o
	2	3
×		4
	1	2
	8	0

	t	o
	2	3
×		4
	1	2
+	8	0
	9	2

23 x 4 = 92

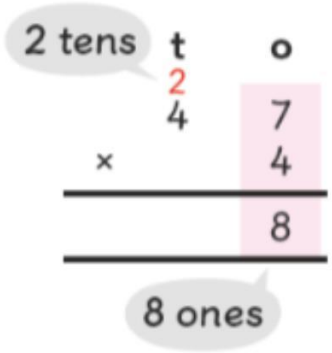
47 x 4 =



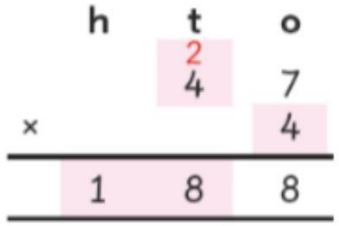
Regrouping (moving to compact method):

47 x 4 =

Multiply the Ones (and exchange surplus Ones for Tens)



Multiply the tens by 4.

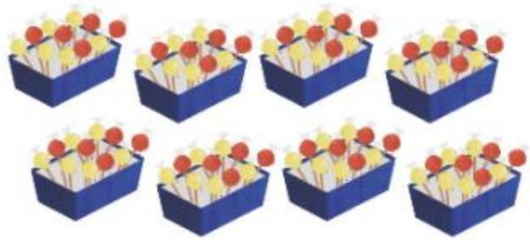
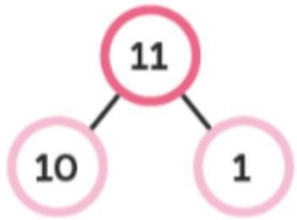
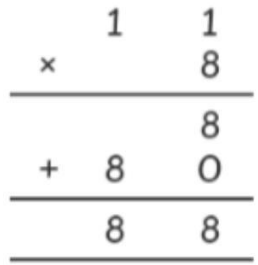
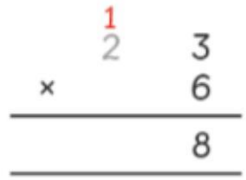


		Children use symbols to stand for unknown numbers to complete equations using inverse operations: □ x 5 = 20 3 x △ = 18 □ x ○ = 32 Partitioning: 38 x 5 = (30 x 5) + (8 x 5) = 150 + 40 = 190 Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

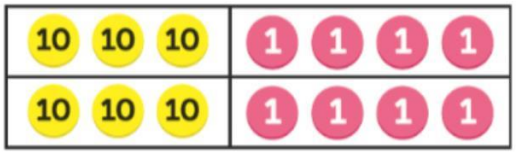
YEAR 4 - MULTIPLICATION

End of Year Objective:

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout and using all multiplication tables up to 12x12

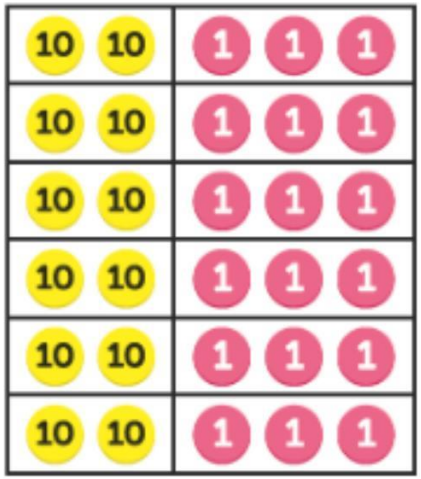
CONCRETE	PICTORIAL	ABSTRACT
Children use key facts to scale up with multiples of 10: 6 x 2 = 12 6 x 20 = 6 x 20 Tens = 120 Tens Use place value counters to visualise: 11 x 8 =  	 Using the part whole model to partition to multiply:  8 × 11 = 80 + 8 = ☐	Multiply the Ones then multiply the Tens:  **Regrouping:** Multiply the Ones:  Multiply the Tens: 

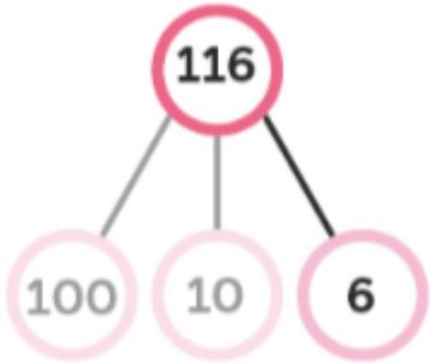
34 x 2 =



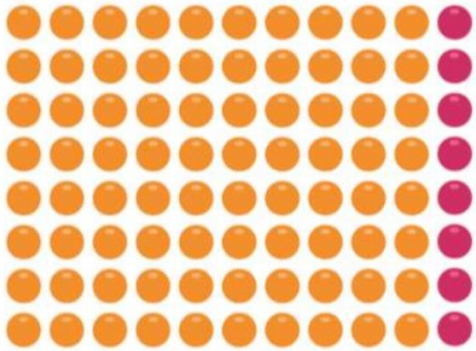
Regrouping:

23 x 6=





Using arrays:



Children use the bar model:

23 x 3 =

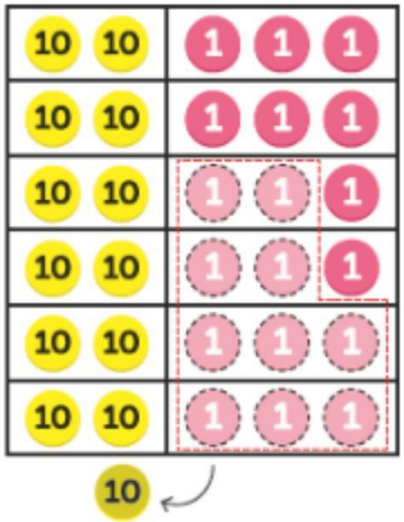
$$\begin{array}{r} {\scriptstyle 1} \\ 23 \\ \times\ 6 \\ \hline 138 \end{array}$$

Multiply the Ones
Multiply the Tens
Multiply the Hundreds
Add them all

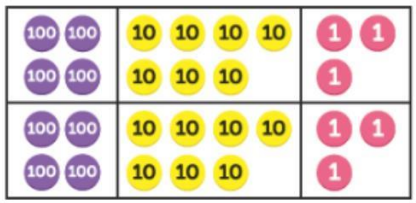
$$\begin{array}{r} 123 \\ \times\ 3 \\ \hline 9 \\ 60 \\ +\ 300 \\ \hline 369 \end{array}$$

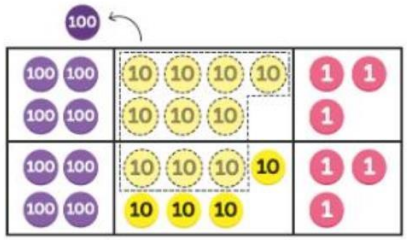
Regrouping:

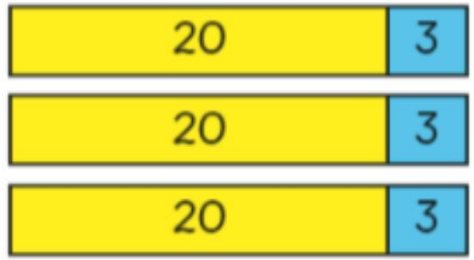
$$\begin{array}{r} {\scriptstyle 3} \\ 16 \\ \times\ 6 \\ \hline 6 \end{array}$$



473 x 2 =



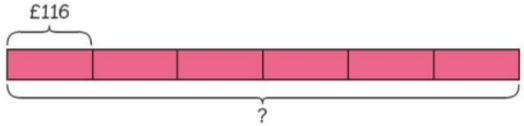


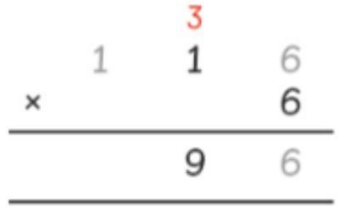


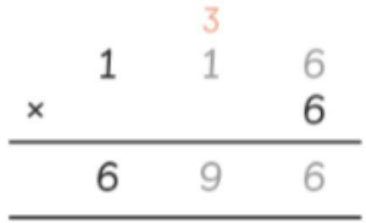
20 x 3 = 60
3 x 3 = 9
60 + 9 = 69

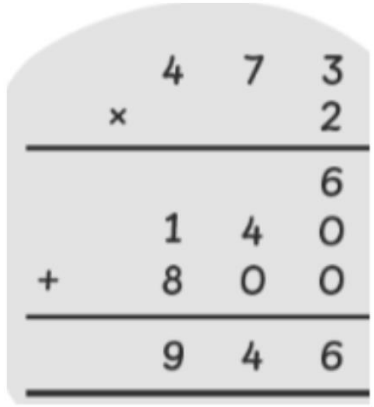
123 x 3 =
100 x 3 = 300
20 x 3 = 60
3 x 3 = 9
300 + 60 + 9= 369

116 x 6 =

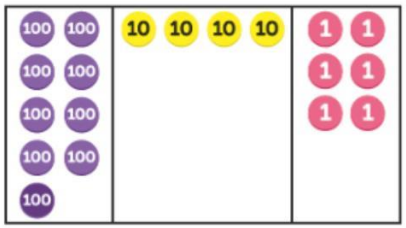






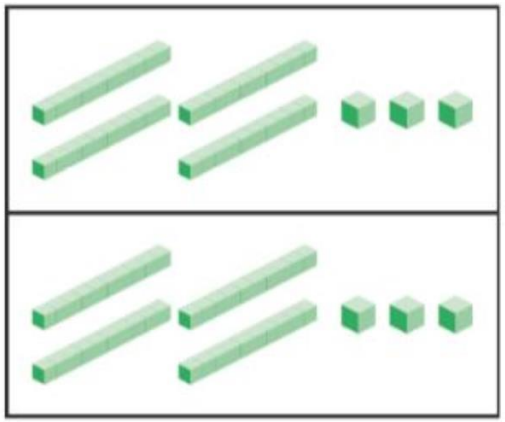


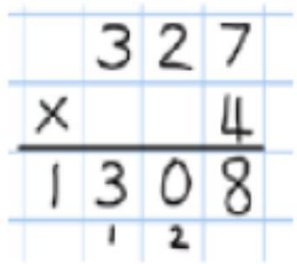
When confident, children move to the compact method:



Using Base Ten:

43 x 2 =





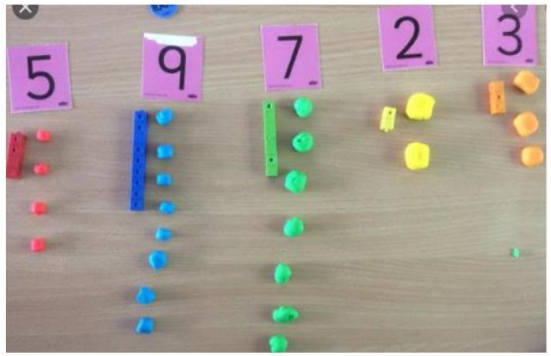
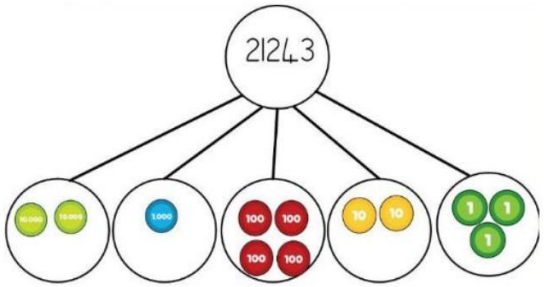
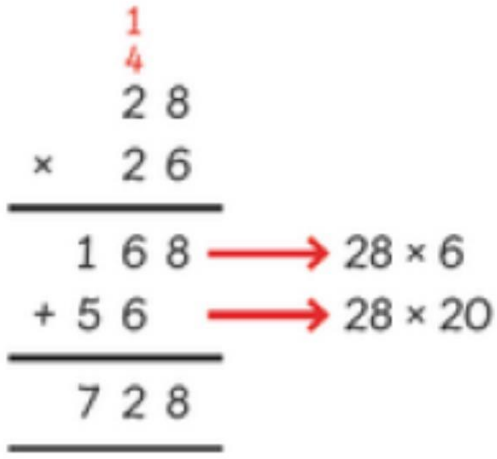
When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

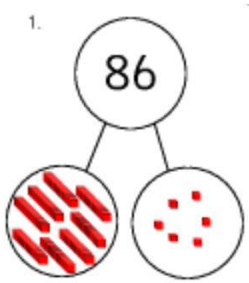
Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

YEAR 5 - MULTIPLICATION ×

End of Year Objective:

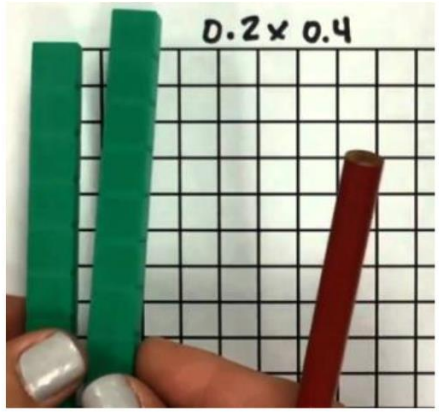
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

CONCRETE	PICTORIAL	ABSTRACT
Once the children are confident at multiplying a 2-digit and 3-digit numbers by a 1-digit number and have been given the precious concrete and pictorial experiences most children will not need the concrete and pictorial approach. It is important that children see the value of partitioning the number: 	Part whole helps with partitioning to see the multiplication of each place value:  Partitioning within a grid can also be helpful:	 Multiply by the Ones Multiply by the Tens

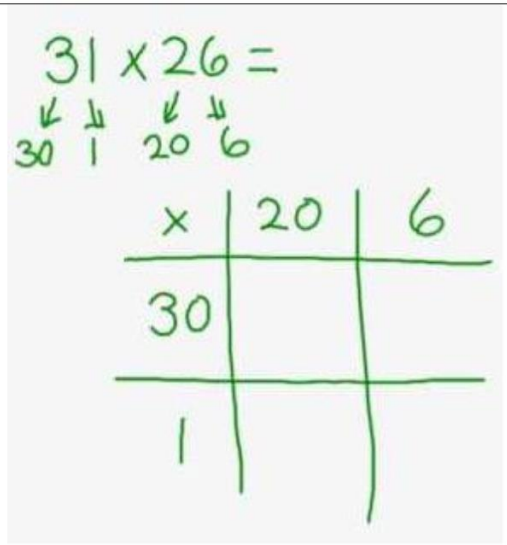


For decimals:

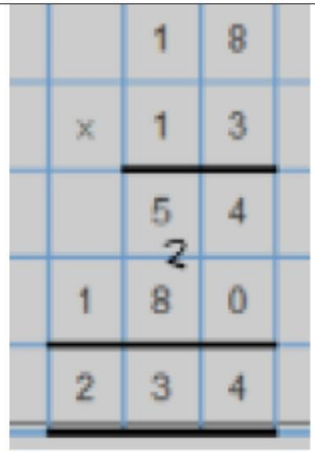
Base Ten can help reinforce the idea of place value beyond the decimal point:



And also arrange the calculation:



×	200	80	1
20			
3			

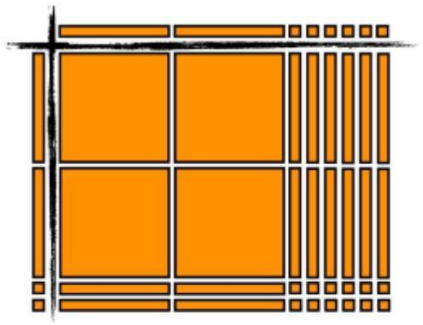


8 x 3 = 24

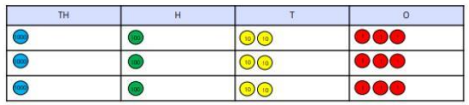
Carry the 2 of the Tens, then do 10 x 3= 30 and add the 20= 50.

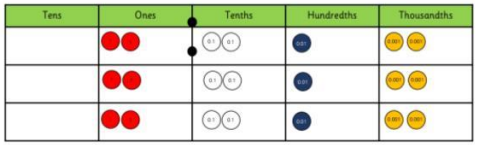
Then multiply the 8 of the Ones by the 0 Ones of the Ten of the 13.

$$\begin{array}{r} {\scriptstyle 1} \\ 22 \\ \times\ 26 \\ \hline 132 \\ +\ 440 \\ \hline 572 \end{array}$$



22 x 26 =





$3.26 \times 4 =$

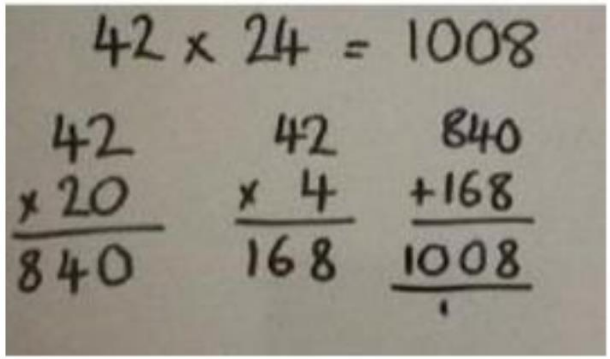
$3 \quad 0.2 \quad 0.06$

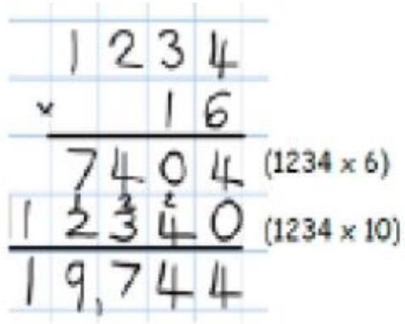
$(4 \times 3) + (4 \times 0.2) + (4 \times 0.06)$

	3	0.2	0.06
4	12	0.8	0.24

12.00 + 0.8 + 0.24 = 13.04

Support may be given through different calculations:



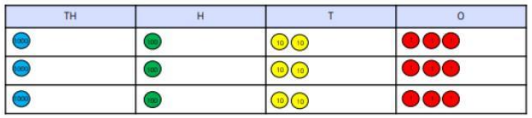
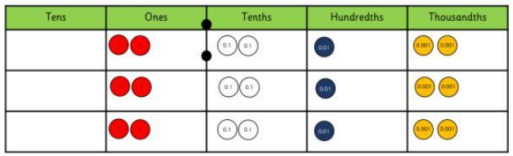
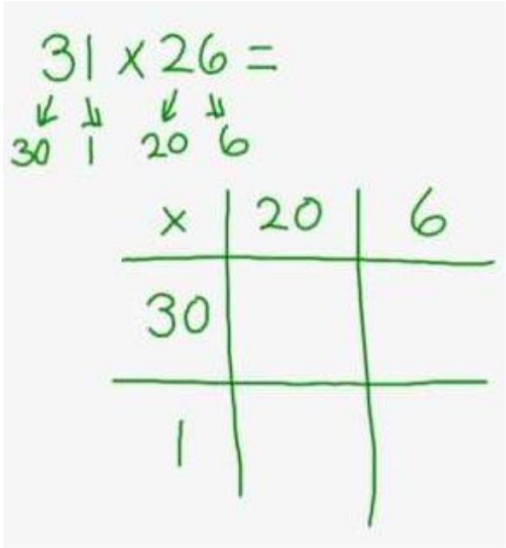
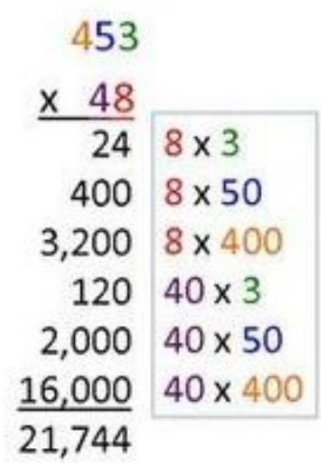


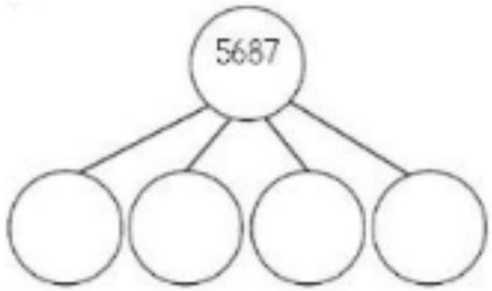
YEAR 6 - MULTIPLICATION

End of Year Objective:

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.

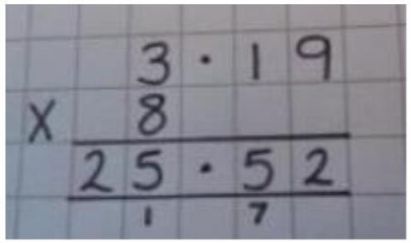
Multiply one-digit numbers with up to two decimal places by whole numbers.

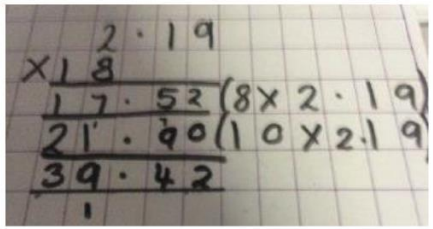
CONCRETE	PICTORIAL	ABSTRACT
Once the children are confident at multiplying a 2-digit and 3-digit numbers by a 1-digit number and have been given the precious concrete and pictorial experiences most children will not need the concrete and pictorial approach. Place value counters can support understanding:   	Partitioning can help the process of multiplication:  	Multiply the Ones first; Multiply the Tens Multiply the Hundreds Multiply the Thousands Find the total Option for zero as a place holder 



		3	5	6	1
	×			2	4
	1	4_{2}	2_{2}	4	4
+	7_{1}	1_{1}	2	2	
	8	5	4	6	4

When multiplying decimals, use same method but ensure decimal point is in with all values carefully written, in line, on either side.





		Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.