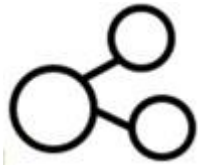
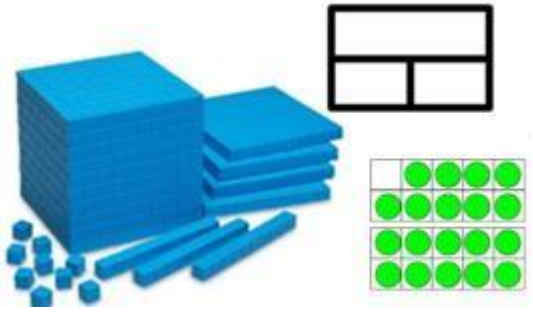
Calculation Policy

A D D I T I O N

Including Models and Images





Reviewed October 2022

“The richest concept images will allow children to make the most effective numerical connections, enabling them to communicate mathematically.”

Introduction

This policy exemplifies a recommended progression through both mental and written calculations for the four operations and has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of Mathematics. There is a strong focus on the use of models and images to support children's concept image of number and their understanding of how this relates to methods of calculation. It is also designed to give pupils a consistent and smooth progression of learning in calculations across the school, taking into account Maths No Problem! : a Singaporean teaching style in Maths.

The Calculation Policy shows methods that pupils will be taught within their respective year group. It is shown in teaching order. Children should be confident in choosing and using a strategy that they know will get them to the correct answer as efficiently as possible; pupils are free to choose their preferred method to solve calculations. Although our policy is set out based on The National Curriculum year group expectations, children work through the calculation policy systematically. Some children may therefore be working below year group expectation and should be taught the method appropriate for their individual stage in learning. This process should not be rushed; children should be moved on when they are ready.
"Children develop/learn in different ways and at different rates" – EYFS Principles.

Up to Year 3 the main emphasis should be on children working practically and mentally and recording through jottings. Once written methods are introduced, using practical images to support and develop mathematical understanding, mental skills must be kept sharp by continuing to develop and apply them with appropriate examples.

Should children be taught one standard method for each operation?

Children should work through the school's agreed progression in methods in order that they know and understand a compact standard method for each numerical operation by the end of Year 6.

How can children's readiness for written calculations be judged?

Judgements will need to be made as to whether pupils possess sufficient of these skills to progress. Different prerequisite skills are needed for each operation.

A short list of criteria for readiness for written methods of addition and subtraction would include:

- Do children know addition facts to 20?
- Do they understand place value and can they partition numbers into hundreds, tens and units?
- Do they use and apply commutative and associative laws of addition?
- Can they add at least three 1-digit numbers mentally?
- Can they add and subtract any pair of 2-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Corresponding criteria to indicate readiness to learn written methods for multiplication and division are:

- Do the children know their 2,3,4,5 and 10 times tables and corresponding division facts?
- Do they know the result of multiplying by 0 or 1?
- Do they understand 0 as a place holder?
- Can they multiply 2 and 3 digits mentally by 10 and 100?
- Can they use their knowledge of all the multiplication tables to approximate?
- Can they find products using multiples of 10?
- Do they use the commutative and associative laws of multiplication?
- Can they halve and double 2-digit numbers mentally?
- Can they use multiplication facts to derive mentally, other multiplication facts they don't know?
- Can they explain their mental strategies orally and record them using informal jottings?

Concrete, Pictorial, Abstract:

A key principle behind the Singapore Maths textbooks and Maths Mastery is based on the concrete, visual and abstract approach. Pupils are first introduced to an idea or skill by acting it out with real objects (a hands-on approach). Pupils then are moved onto the visual stage, where pupils are encouraged to relate the concrete understanding to pictorial representations. The final abstract stage is a change for pupils to represent problems by using mathematical notation.

Whilst this calculation policy aims to show the CPA approach to the different calculations, it is not always noted further up the year groups. However, it is expected that the CPA approach is used continuously in all new learning and calculations even when not noted.

Reasoning and Problem Solving:

Once children are fluent in the calculation strategy for their year group, we deepen and embed understanding through providing children with a range of reasoning and problem solving skills that allow the children to show their full understanding in a range of different context.

Monitoring of Written Calculations

It is important that procedures are in place to ensure that all staff are aware of the progression through calculations, and that children are being taught appropriate methods for their age and ability, which are in line with the agreed policy. This may include book sampling, reflective enquiries, monitoring of planning, learning walks and pupil interviews.

Progression for Addition

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of subtraction
- commutative i.e. 5 + 3 = 3 + 5
- associative i.e. 5 + 3 + 7 = 5 + (3 + 7)

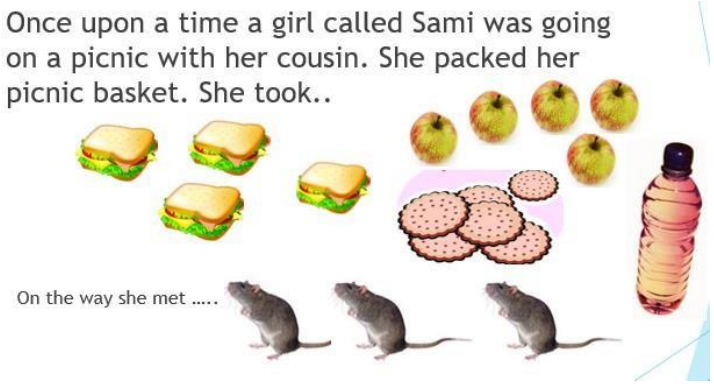
The fact that it is commutative and associative means that calculations can be rearranged, e.g.
4 + 13 = 17 is the same as 13 + 4 = 17.

In **EYFS** pupils should be developing their concept of the number system through the use of concrete materials and pictorial representations. They should experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc. It is vital to develop a deep number sense so Number Talk is really important!

NURSERY

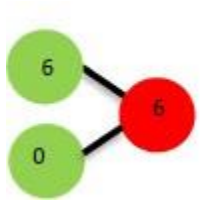
NUMBERS AND COUNTING

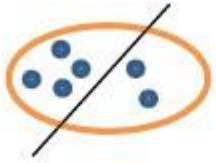
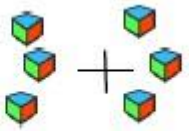
Stories and rhymes are cross curricular, encourage rhythm, develop language and vocabulary, encourage imagination and develop understanding. Maths story maps and trails also aid progression:



- Oral counting is a child's first experience of number and mathematics.
- Making connections between saying the number names and counting objects is the **first step** towards children's understanding of the number system
- Counting is one tool for building up calculation strategies
- We need to count backwards is as well as forwards.

Pupils should recognise **different ways of making numbers**. E.g 6 can be made as:



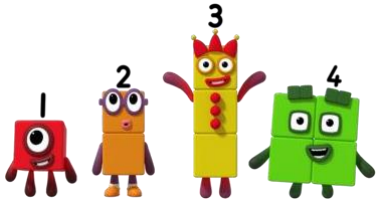

Linking number names, numerals and value is so important!

One-to-one correspondence refers to the ability to match one object to one number consistently. Without this skill, children will not be able to understand how many objects are in a group (the cardinal value).

Understanding the 'cardinality principle' means that a child appreciates that the last number counted and indicates how many things are in the set.

'Subitising' is knowing how many without counting. Patterns are integral to this ability in order to 'see' numbers in sets, e.g. the patterns on a die, dominoes and fingers



Conservation of number means that children need the opportunity to recognise amounts that have been rearranged and to generalise that, if nothing has been added or taken away, then the amount is the same.

Comparison of sets is explored -one set may be more than, less that or equal to another. Children need to have the opportunity to match a number symbol with a number of things. There are opportunities to have a range of number symbols available, e.g. wooden numerals, play dough, calculators, handwritten (include different examples of a number, e.g. , ,).

EARLY LEARNING GOAL:

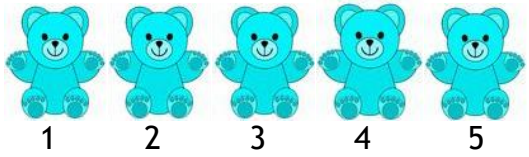
Using quantities and objects, children add **two** single-digit numbers and **count on** to find the answer.

It is essential that children have a solid understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order. They should be able to confidently subitise (recognise quantities without counting) up to 5 and can automatically recall number bonds for numbers 0-5 and for 10, including corresponding partitioning facts.

Counting all method

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating **3 + 2**, they are encouraged to count out three bears and count out two bears.

To find how many altogether, children touch and place them in a line one at a time whilst counting.



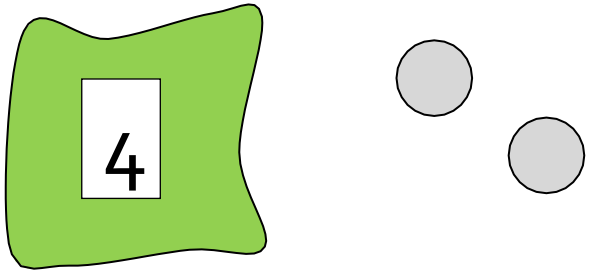
This method allows the children to keep track of what they have already counted to ensure they don't count the same item twice.

Counting on method

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted.

For example, when calculating **4 + 2**, count out the two groups of counters as before.

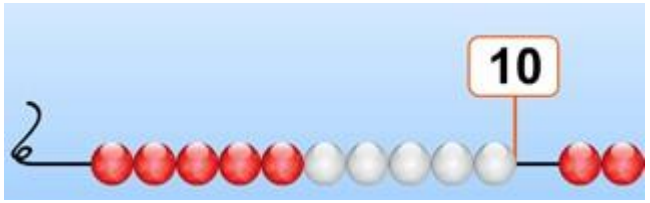
then cover up the larger group with a cloth.



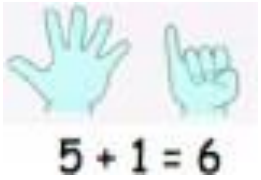
For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before.

Children use NUMICON to find one more.

BEAD STRINGS or BARS can be used to illustrate addition

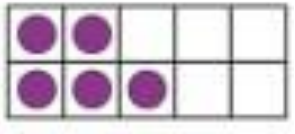


Children will begin to solve simple word moving a toy or whole self.
They will begin to find **one more to ten**.



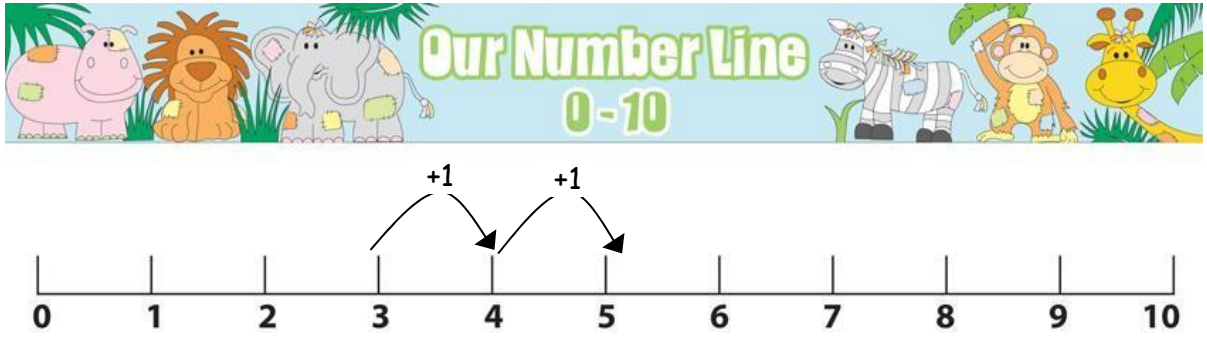
They will use a NUMBER TRACK to count on, either problems using their fingers.

TEN FRAME



When appropriate, teachers *begin to demonstrate* the use of the number line.

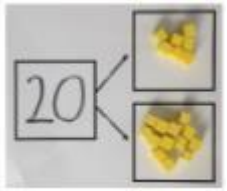
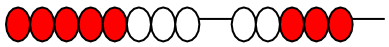
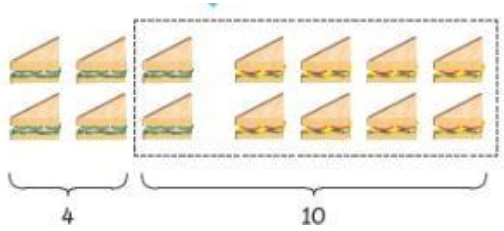
3 + 2 = 5



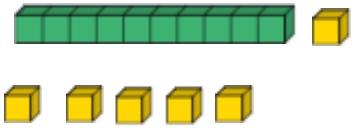
YEAR 1- ADDITION

End of Year objective:

Add one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

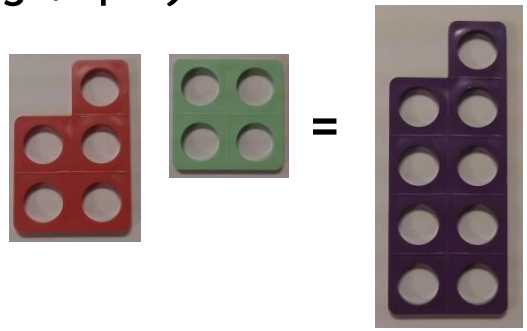
CONCRETE	PICTORIAL	ABSTRACT
Children will continue to use practical equipment, combining groups of objects to find the total by counting all or counting on.  Making numbers to 20   Using their developing understanding of place value, they will move on to be able to use Base 10 and Numicon equipment to make teens numbers using separate tens and ones. **Using Base Ten:**	Teachers and children will use images for objects:  and the idea is to navigate children to 'notice' partitioning a number, then bonding to make ten.  Part part whole and also the ten frame will be used:	Children will use their knowledge of number to calculate and record written calculations e.g. 12 + 6 = 18 Children should be taught that the = sign does not always come at the end of the calculation: 20 = 19 + 1 8 + 12= 20

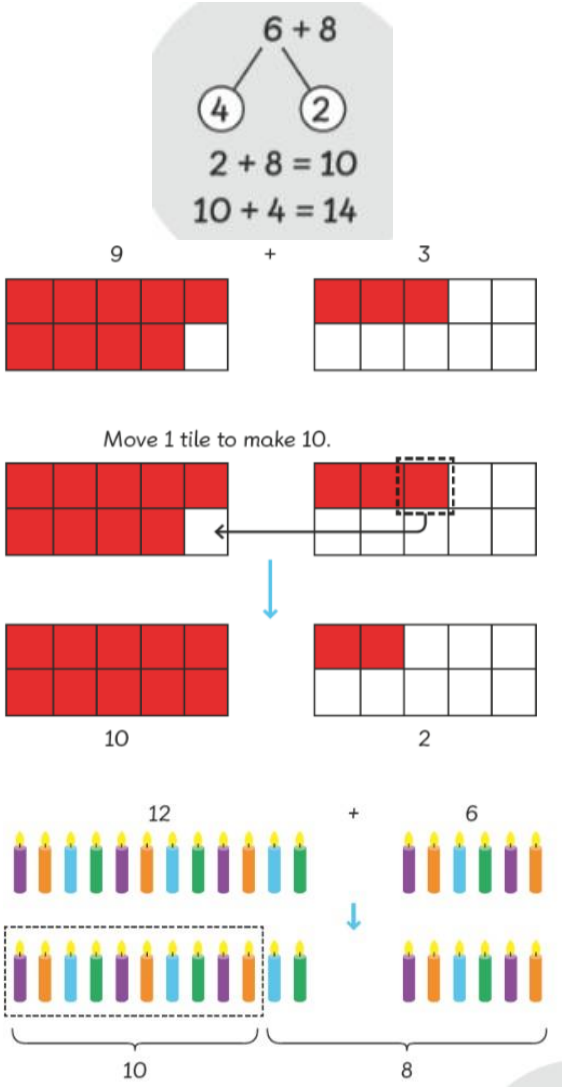
For example, when adding 11 and 5, they can make the 11 using a ten rod and a unit.
So 11 + 5 = 16.
If possible, they should use two different colours of base 10 equipment so that the initial amounts can still be seen.



Using Numicon:

5 + 4 = 9

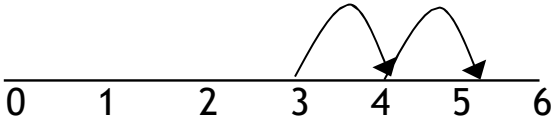




The bar model can also be drawn:

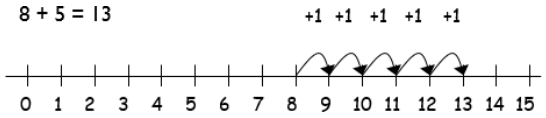
Teachers demonstrate the use of the numberline, starting with the largest number first:

3 + 2 = 5



Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

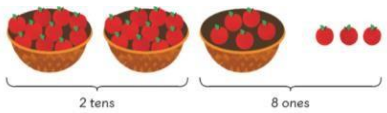
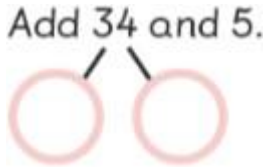
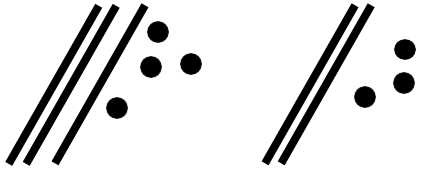
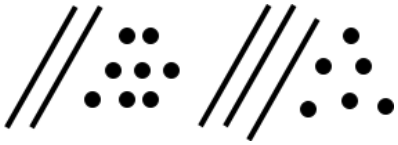
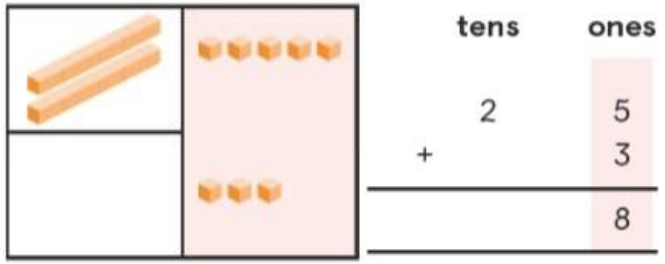
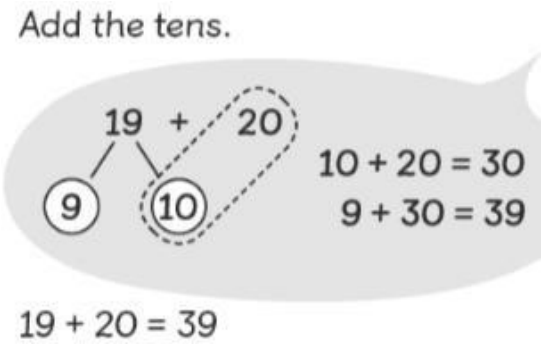
8 + 5 = 13

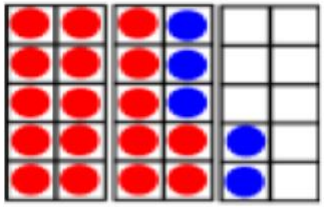


YEAR 2- ADDITION

End of Year objective:

Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one – digit numbers.

CONCRETE	PICTORIAL	ABSTRACT
Children continue to use practical equipment including counters and tens frames, multi-link cubes, Base 10 apparatus, straws, bead strings ,Numicon along with part part whole to show how to add the numbers. Addition will be represented in real life:  How many are there altogether?  **17+5 = 22** Children use the ten frame to make 'magic 10'	Children use pictorial representations including drawings and images of physical apparatus, as well as the bar model, part whole model and number lines. Children can record their drawings of Base 10 using lines for the rods and dots for the ones blocks. e.g. 34 + 23 =  With exchange: e.g. 28 + 36 = 	Children will begin to use the formal column method. Intitially without crossing through a tens then progressing to crossing through a tens by carrying.  Adding the tens: 

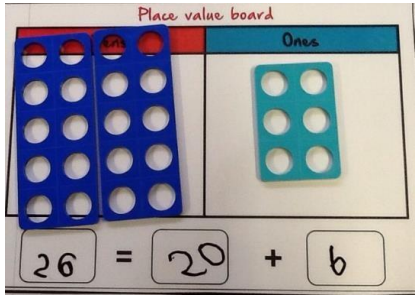


Children start to explore the pattern that if:

17+5 = 22

27 + 5 = 32

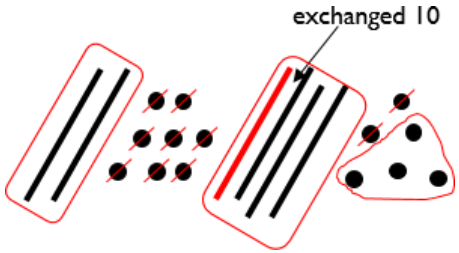
Children can use a place value grid to combine add the physical apparatus in various forms:



Base Ten:

32 + 21 = 53

Will become:

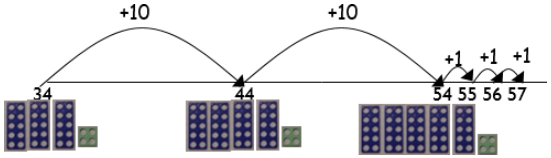


It is important that children circle the remaining tens and ones after exchange to identify the amount remaining.

Number lines:

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.
First counting on in tens and ones.

34 + 23 = 57



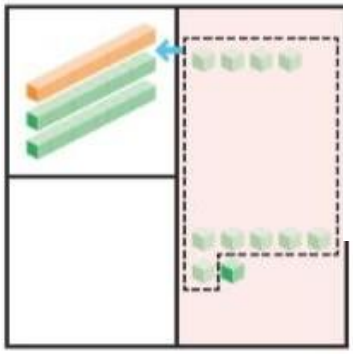
Adding with renaming:

Add the ones.

4 ones + 7 ones = 11 ones

Regroup the ones.

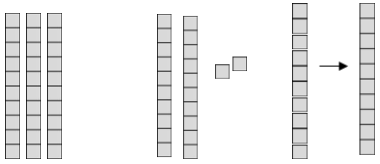
11 ones = 1 ten and 1 one



	tens	ones
	2	4
+		7
	1	1

When the ones total more than 10, children should be encouraged to exchange 10 ones for 1 ten. This is the start of children understanding 'carrying' in vertical addition. For example, when calculating
35 + 27, they can represent the amounts using Base 10 as shown:

Then, identifying the fact that there are enough units/ones to exchange for a ten, they can carry out this exchange:



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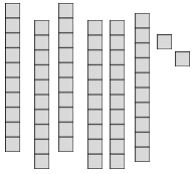
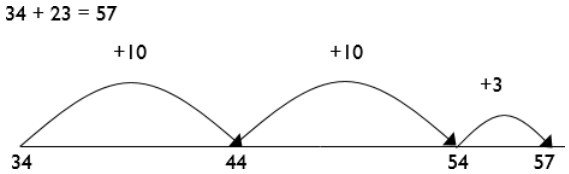


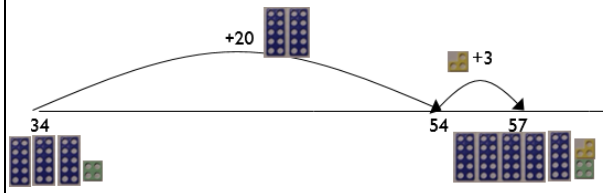
This can also be done with Numicon.

Then helping children to become more efficient by adding the **ones in one jump** (by using the known fact 4 + 3 = 7).



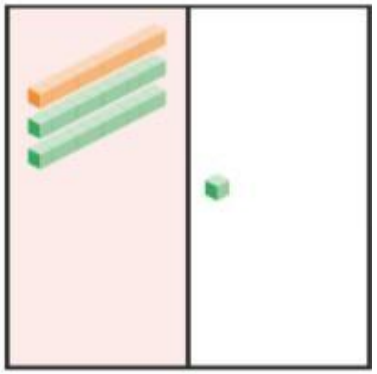
Followed by adding the **tens in one jump** and the **ones in one jump.**

34 + 23 = 57



Add the tens.

1 ten + 2 tens = 3 tens



	tens	ones
	2	4
+		7
	1	1
+	2	0
	3	1

24 + 7 = 31

YEAR 3- ADDITION

End of Year objective:

Add numbers with up to three digits, using formal written methods of columnar addition.

*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

CONCRETE

Children will build on their knowledge of using Base 10 equipment from Year 2 and continue to use the idea of exchange.
Children should add the least significant digits first (i.e. start with the ones), and in an identical method to that from Year 2, should identify whether there are greater than ten Ones which can be exchanged for one Ten.
Numicon and counters to explore adding numbers by carrying:

Without renaming
Adding Ones:

PICTORIAL

Children use pictorial representations including drawings and images of physical apparatus, as well as the bar model, part whole model and number lines.

ABSTRACT

They can use a place value grid to begin to set the calculation out vertically and to support their knowledge of exchange between columns

Column method (expanded):

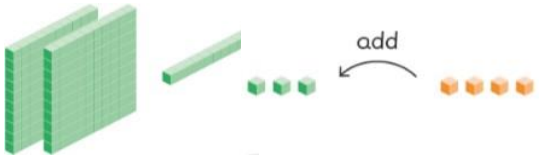
236 + 8=

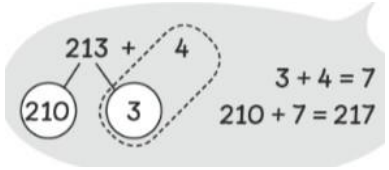
Add the Ones first:

	h	t	o
			8
+	2	3	6
		1	4

Then add the Tens:

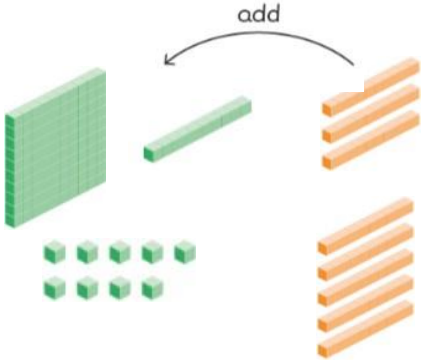
	h	t	o
			8
+	2	3	6
		1	4
+		3	0

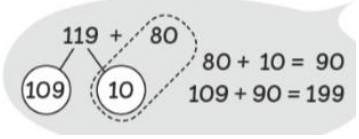




213 + 4 = 217

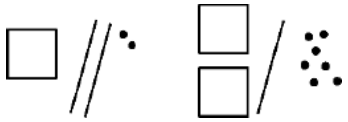
119 + 80 =
Add the Tens





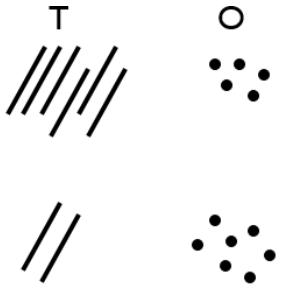

```
  5 3 1
+ 1 4 3
-------
  6 7 4
```

Children can record their drawings of Base 10 using squares for the 100s block, slanted lines for the rods and dots for the ones blocks.



65 + 27=

STEP 1:



Add the Hundreds:

	h	t	o
			8
+	2	3	6
		1	4
		3	0
+	2	0	0
	2	4	4

TO

```
  67
+ 24
  11  ( 7 + 4)
  80  (60 + 20)
  91
```

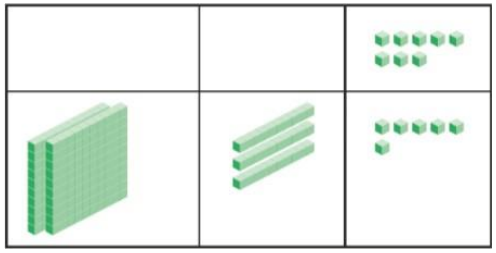
HTO

```
  267
+  85
   12  ( 7 + 5)
  140  (60 + 80)
  200
  352
```

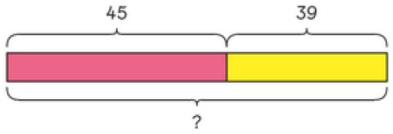
Moving to Column method (compact):

With renaming

236 + 8 =



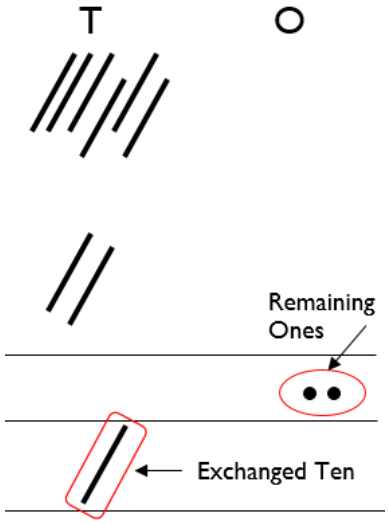
Children continue to use bars as a visual model to solve addition calculations and exposed to word problems.



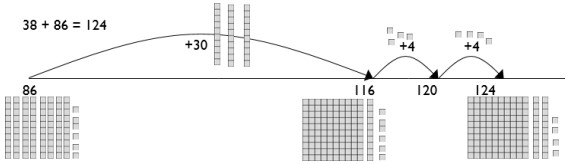
STEP 2:

Add the Ones 5 + 7= 12

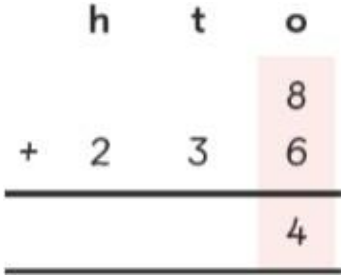
EXCHANGE the 12 Ones for one Ten and 2 Ones



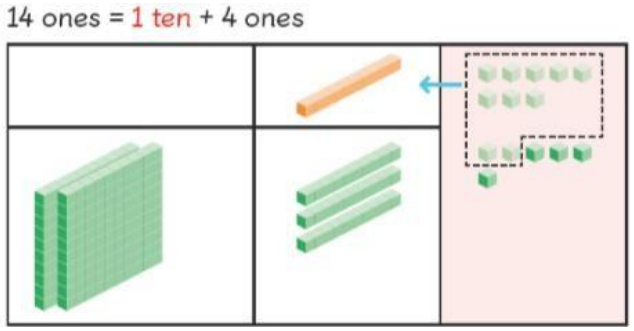
Children will continue to use empty number lines with increasingly large numbers.
Count on from the largest number irrespective of the order of the calculation.



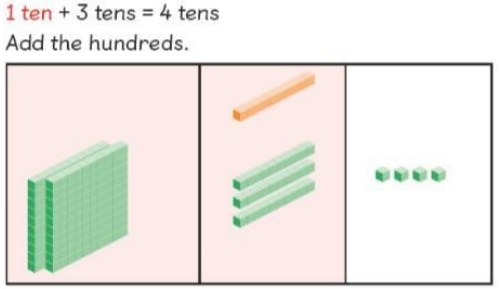
Add the Ones:

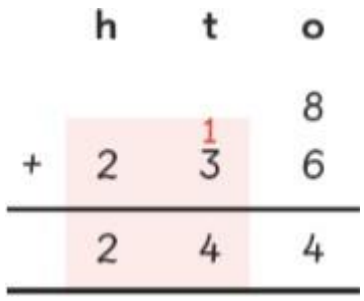


Then regroup them:



Add the Tens:



		Add the Hundreds: 

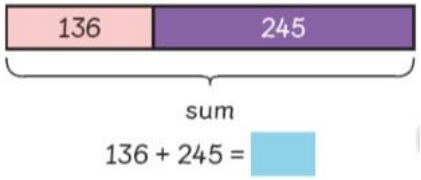
YEAR 4 - ADDITION

End of Year objective:

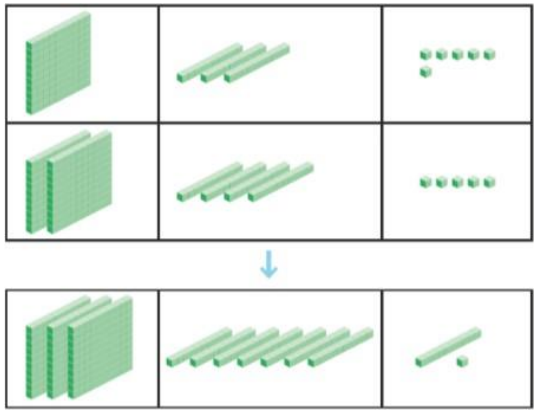
Add numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar addition where appropriate.

CONCRETE	PICTORIAL	ABSTRACT
Children continue to use Base 10 and place value counters to add, exchanging 10 Ones for a ten and ten Tens for 100 and ten Hundreds for 1000. Bar model can beused:	Children use pictorial representations including drawings and images of physical apparatus, as well as the bar model, part whole model and number lines.	**Adding without renaming** Column method:

Find the sum of 136 and 245.



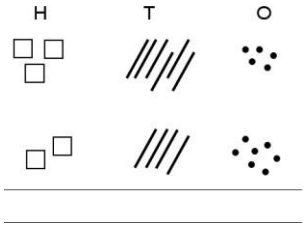
136 + 245 = 381



Use place value counters:
136 + 245 = 381

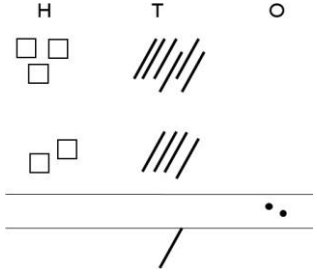
As in previous year groups, children can record their drawings of Base 10 using squares for the 100s block, slanted lines for the rods and dots for the Ones blocks.

STEP 1: 365+247=

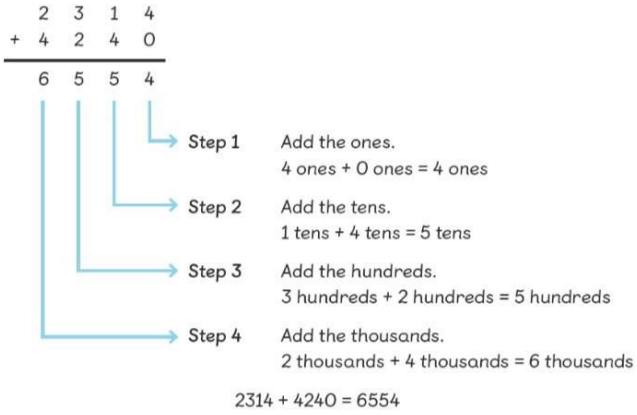


	H	T	O
	3	6	5
+	2	4	7

STEP 2: Add the Ones



	H	T	O
	3	6	5
+	2	4	7
			2
		1	

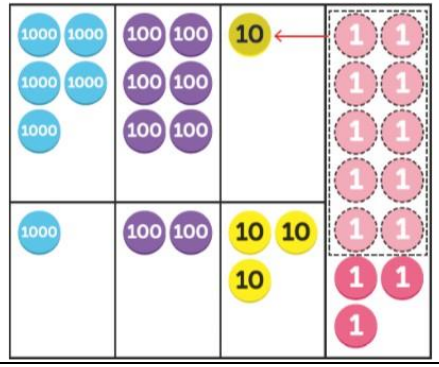


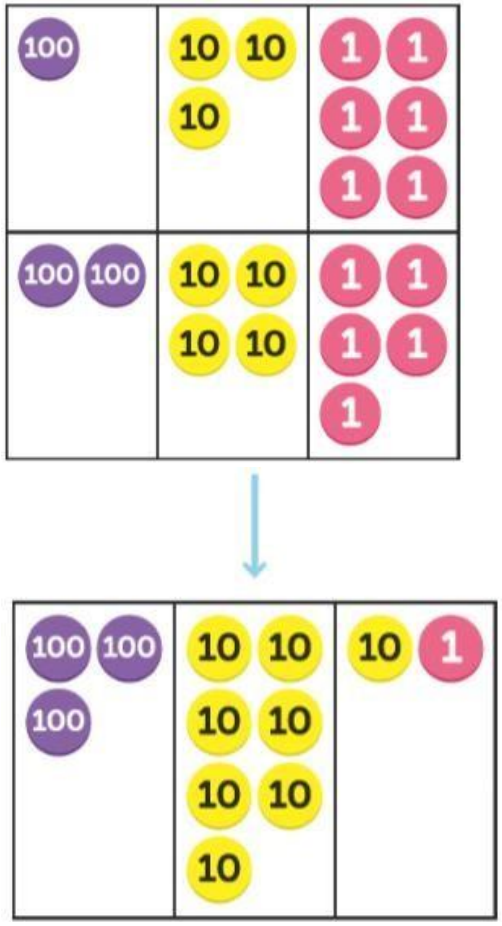
Adding with renaming:

Find the sum of 5608 and 1235

STEP 1: Add the Ones (Expanded)

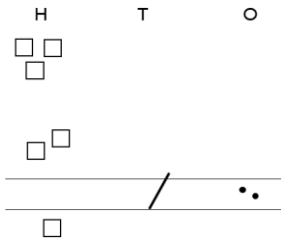
8 Ones + 5 Ones= 13 Ones
RENAME the Ones as:
1 Ten and 3 Ones





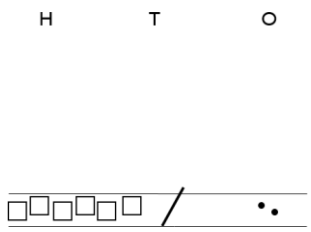
Enable children to explore decimals with place value counters and Numicon:

STEP 3: Add the Tens



	H	T	O
	3	6	5
+	2	4	7
		1	2
	1	1	

STEP 4: Add the Hundreds



	H	T	O
	3	6	5
+	2	4	7
	6	1	2
	1	1	

Add 5608 and 1235.

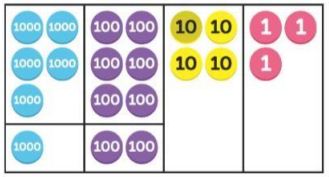
	5	6	0	8
+	1	2	3	5
			1	3
			3	0
		8	0	0
+	6	0	0	0

add ones
add tens
add hundreds
add thousands

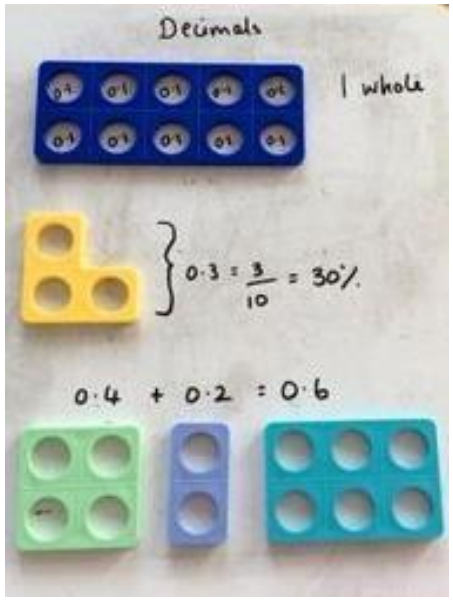
Compact method:

			1	
	5	6	0	8
+	1	2	3	5
				3

STEP 2: Add the Tens

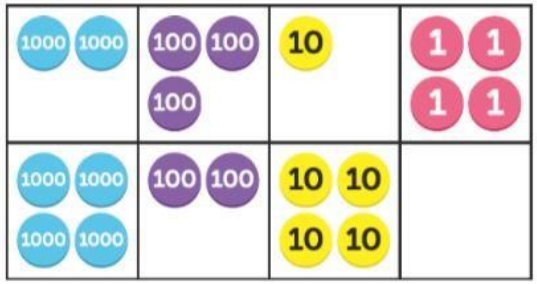


			1	
	5	6	0	8
+	1	2	3	5
			4	3

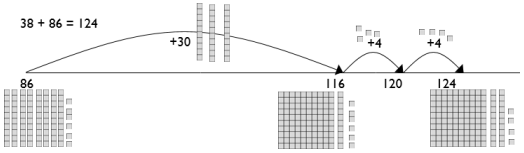


Also use place value counters:

Find the sum of 2314 and 4240.



Also using the number line:



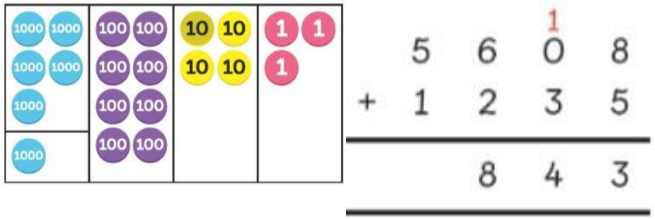
38 + 86= 124

Start with the largest number

Add the Tens

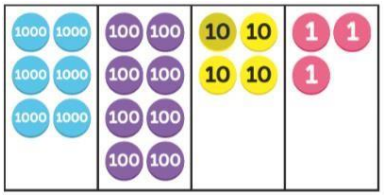
Add the Ones

STEP 3: Add the Hundreds



$$\begin{array}{r} {\scriptstyle 1} \\ 5\,6\,0\,8 \\ +\;1\,2\,3\,5 \\ \hline 8\,4\,3 \end{array}$$

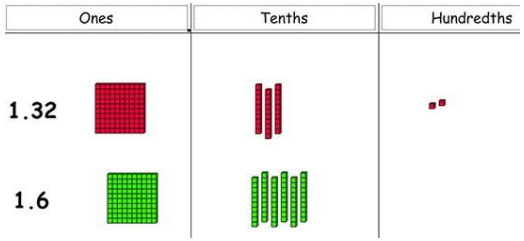
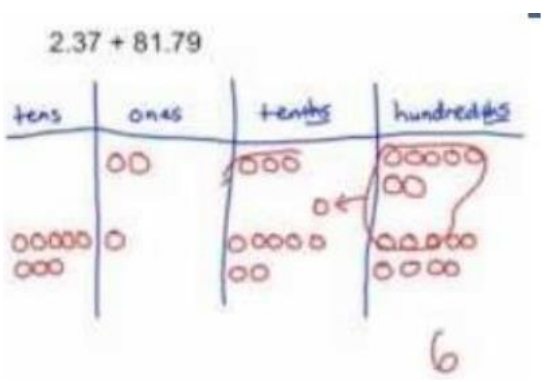
STEP 4: Add the Thousands

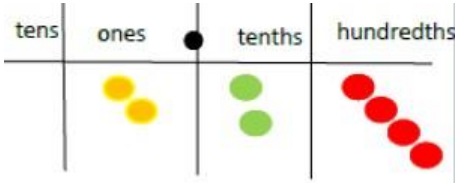


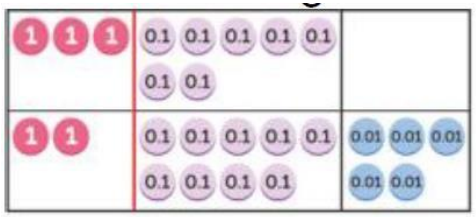
YEAR 5 - ADDITION

End of Year objective:

Add whole numbers with more than 4 digits *and decimals with two decimal places*, including formal written methods (columnar addition).

CONCRETE	PICTORIAL	ABSTRACT
Children working at a Year 5 level should be secure in the use of column and will not necessarily need to use Base 10 as in previous years as their understanding of the concept should be secure. However, they should be given, planned purposeful opportunities to use place value counters and Base Ten to explore addition of decimals. 	Children can draw visual representations of place value counters to support the transition from concrete to abstract. 	Children should be given opportunities to add: several numbers with different numbers of digits, understanding the place value decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another. amounts of money and measures, including those where they have to initially convert from one unit to another. Children should continue to use the carrying method to solve calculations such as: 3 3 6 4 + 2 4 7 3 6 1 1 1 1





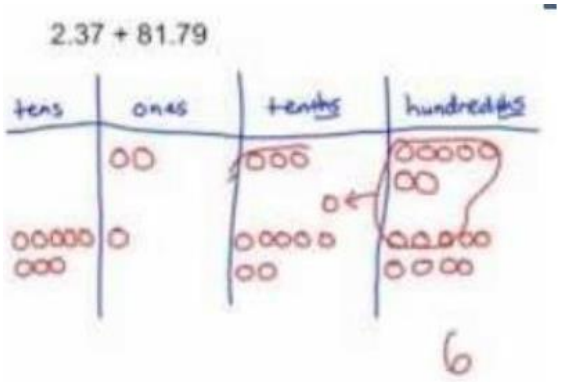
$$\begin{array}{r} 3\,1\,2\,1 \\ 3\,7 \\ +\quad 1\,4\,8 \\ \hline 3\,3\,0\,6 \\ \hline 1\,1 \end{array}$$

$$\begin{array}{r} 3.56 \\ +\,2.47 \\ \hline 6.03 \\ \hline 1 \end{array}$$

YEAR 6 - ADDITION

End of Year objective:

Add whole numbers and decimals using formal written methods (columnar addition).

CONCRETE	PICTORIAL	ABSTRACT
Children working at a Year 6 level should be secure in the use of column and will not need to use Base 10 as in previous years as their understanding of the concept should be secure.	Children can draw visual representations of place value counters to support the transition from concrete to abstract. 	Children will be adding: Several numbers with different numbers of digits, understanding the place value. Decimals with up to two decimal places (with mixed numbers of decimal places), knowing that decimal points line up after one another. Amounts of money and measures, incldluing those where they have to intially convert from one unit to another. Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

		4 2 6 4 3 2 7 8 6 3 + 4 6 8 1 1 1 9 4 4 1 1 2 1 4 0 1 . 2 0 2 6 . 8 5 + 0 . 7 1 4 2 8 . 7 6 1 When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.